Consistency of JP, P and S.H Janus anti-gravity theories

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We examine consistency of the predictions within JPP Janus anti-gravity theory as for the emission of gravitational waves.

0.1. Gravitational Waves

We follow the method and notations of S Weinberg Gravitation and cosmology section 7.6. The two metrics of the Janus theory written as usual in the asymptotically Minkowskian form suitable to compute the emission rate of gravitational waves by an accelerating body are:

\[ g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \]  

\[ g^*_{\mu\nu} = \eta_{\mu\nu} + h^*_{\mu\nu} \]  

The linearized JPP equations read

\[ R^{(1)}_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} R^{(1)\lambda}_{\lambda} = -8\pi G (T_{\mu\nu} - T^*_{\mu\nu} + t_{\mu\nu}) \]  

\[ R^{* (1)}_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} R^{* (1)\lambda}_{\lambda} = -8\pi G (T^*_{\mu\nu} - T_{\mu\nu} + t^*_{\mu\nu}) \]  

where we have isolated the nonlinear terms in \( h \) and \( h^* \) and put them in \( t \) and \( t^* \) on the right side of the equations. The usual interpretation is that not only the matter content of \( T \) and \( T^* \) terms source the gravitational field \( h \) and \( h^* \) but also the energy momentum of the gravitational fields themselves in \( t \) and \( t^* \).

We are here only interested in the gravitational waves emitted by a body in \( g_{\mu\nu} \) which only contributes to the energy momentum tensor \( T_{\mu\nu} \), so neglecting \( T^* \) terms,

\[ R^{(1)}_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} R^{(1)\lambda}_{\lambda} = -8\pi G (T_{\mu\nu} + t_{\mu\nu}) \]  

\[ R^{* (1)}_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} R^{* (1)\lambda}_{\lambda} = -8\pi G (-T_{\mu\nu} + t^*_{\mu\nu}) \]  

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Neglecting the self energy contribution in the weak field approximation leads to very familiar equations for the generation of $h$ and $h^*$ (Weinberg 10.1)

\[ \Box h_{\mu\nu} = -16\pi GS_{\mu\nu} \]  
\[ \Box h^*_{\mu\nu} = 16\pi GS_{\mu\nu} \]  

where $S_{\mu\nu} = T_{\mu\nu} - 1/2\eta_{\mu\nu}T$.  

we remember that the linearized Bianchi identities are still obeyed on the left hand sides of Eq 5 6 and it therefore follows the local conservation laws:

\[ \frac{\partial}{\partial x^\mu} (T_{\mu\nu} + t_{\mu\nu}) = 0 \]  
\[ \frac{\partial}{\partial x^\mu} (T_{\mu\nu} - t^*_{\mu\nu}) = 0 \]

Equations 7 and 9 are the starting point to compute the energy momentum radiated by the orbiting body (see Weinberg chapter 10) in General Relativity. The orbit period decay (in-spiraling) was measured in binary pulsars and accurately agree with the GR prediction. This is because the kinetic energy lost by $T_{\mu\nu}$ is carried away by the gravitational wave which energy momentum is $t_{\mu\nu}$ carrying a positive energy.

But now at the same time equations 8 and 10 must also be satisfied. The exact same computation method of Weinberg section 10 now predicts that according those equations, the same body kinetic energy in $T_{\mu\nu}$ should increase as the energy-momentum carried away by the gravitational wave is now $-t^*_{\mu\nu}$ carrying a negative energy. So according those equations the body orbital period is increasing and the body should be out-spiraling.

So we have a couple of equations leading to an inconsistent result relative to the other couple of equations. The problem is the same for Hossenfelder equations in contrast to what we have in 6 where we have a single equation predicting that the body simultaneously radiates positive and negative energy waves and therefore it’s period should remain constant: this is in conflict with what we observe but at least the theory is consistent and it is possible to extend it in order to recover the GR prediction.

Probably the inconsistency of the theory is related to the fact that JPP equations are not derived from an action while the Hossenfelder approach assumes additional intermediary fields (Pull over) which dynamics is not clarified yet must be independent degrees of freedom to insure that the conjugate metrics of the Einstein Hilbert actions can be varied separately in the action extremization procedure.

Also notice that equation alone is just a GR equation with a negative energy source, well known to produce instabilities. Notice that those instabilities have nothing to do with the Bondi run away of a couple of positive and negative masses which is trivially avoided in all Janus theories.
References
   arxiv:0301273