The Dark side of Gravity (living review)

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Dark Gravity (DG) is a background dependent bimetric and semi-classical extension of General Relativity with an anti-gravitational sector. The foundations of the theory are reviewed. The main theoretical achievement of DG is the avoidance of any singularities (both black hole horizon and cosmic initial singularity) and an ideal framework to understand the cancellation of vacuum energy contributions to gravity and solve the old cosmological constant problem. The main testable predictions of DG against GR are on large scales as it provides an acceleration mechanism alternative to the cosmological constant. The detailed confrontation of the theory with SN-Cepheids, CMB and BAO data is presented. The Pioneer effect, MOND phenomenology and Dark Matter are also investigated in the context of this new framework. The Dark Gravity theory is constantly evolving and the latest version of this review is accessible at www.darksideofgravity.com/DG.pdf Latest modifications are in red. Recent modifications are in orange. In blue an alternative DG postulate to be explored.

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1. Introduction

In the seventies, theories with a flat non dynamical background metric and/or implying many kinds of preferred frame effects became momentarily fashionable and Clifford Will has reviewed some of them (Rosen theory, Rastall theory, BSLL theory ...) in his book [35]. Because those attempts were generically roughly conflicting with accurate tests of various versions of the equivalence principle, the flat non dynamical background metric was progressively given up. The Dark Gravity (DG) theory we support here is a remarkable exception as it can easily reproduce most predictions of GR up to Post Newtonian order (as we shall remind in the two following sections) and for this reason deserves much attention since it might call into question the assumption behind most modern theoretical avenues: background independence.

DG follows from a crucial observation: in the presence of a flat non dynamical background $\eta_{\mu\nu}$, it turns out that the usual gravitational field $g_{\mu\nu}$ has a twin, the "inverse" metric $\tilde{g}_{\mu\nu}$. The two being linked by :

$$\tilde{g}_{\mu\nu} = \eta_{\mu\rho}\eta_{\nu\sigma}\left[g^{-1}\right]^{\rho\sigma} = \left[\eta^{\mu\rho}\eta^{\nu\sigma}g_{\rho\sigma}\right]^{-1} \quad (1)$$

are just the two faces of a single field (no new degrees of freedom) that we called a Janus field [3][4][7][14][15]. See also [5][8][9][6] [30][31][32][33][34][28] for alternative approaches to anti-gravity with two metric fields. In the following, fields are labelled with (resp without) a tilde if they are exclusively built from $\tilde{g}_{\mu\nu}$ (resp $g_{\mu\nu}$) and/or it's inverse and/or matter and radiation fields minimally coupled to $\tilde{g}_{\mu\nu}$ (resp $g_{\mu\nu}$). The exceptions are $\eta_{\mu\nu}$ and it's inverse $\eta^{\mu\nu}$.

The action treating our two faces of the Janus field on the same footing is achieved by simply adding to the usual GR and SM (standard model) action, the similar action with $\tilde{g}_{\mu\nu}$ in place of $g_{\mu\nu}$ everywhere.

$$\int d^4x(\sqrt{g}R + \sqrt{\tilde{g}}\tilde{R}) + \int d^4x(\sqrt{g}L + \sqrt{\tilde{g}}\tilde{L}) \tag{2}$$

where R and $\tilde{R}$ are the familiar Ricci scalars respectively built from $g_{\mu\nu}$ and $\tilde{g}_{\mu\nu}$ as usual, L and $\tilde{L}$ the scalar Lagrangians for respectively SM F type fields minimally coupled to $g_{\mu\nu}$ and $\tilde{F}$ fields minimally coupling to $\tilde{g}_{\mu\nu}$ and by convention $g = -det(g_{\mu\nu})$, $\tilde{g} = -det(\tilde{g}_{\mu\nu})$. This theory symmetrizing the roles of $g_{\mu\nu}$ and $\tilde{g}_{\mu\nu}$ is Dark Gravity (DG) and the field equation satisfied by the Janus field derived from the minimization of the action is:

$$\sqrt{g}\eta^{\mu\sigma}g_{\sigma\rho}G^{\rho\nu} - \sqrt{\tilde{g}}\eta^{\nu\sigma}\tilde{g}_{\sigma\rho}\tilde{G}^{\rho\mu} = -8\pi G(\sqrt{g}\eta^{\mu\sigma}g_{\sigma\rho}T^{\rho\nu} - \sqrt{\tilde{g}}\eta^{\nu\sigma}\tilde{g}_{\sigma\rho}\tilde{T}^{\rho\mu}) \tag{3}$$

with $T^{\mu\nu}$ and $\tilde{T}^{\mu\nu}$ the energy momentum tensors for F and $\tilde{F}$ fields respectively and $G^{\mu\nu}$ and $\tilde{G}^{\mu\nu}$ the Einstein tensors (e.g. $G^{\mu\nu} = R^{\mu\nu} - 1/2g^{\mu\nu}R$). Of course from the Action extremization with respect to $g_{\mu\nu}$ (see the detailed computation in the Annex), we first obtained an equation for the dynamical field $g_{\mu\nu}$ in presence of the non dynamical $\eta_{\mu\nu}$. Then $\tilde{g}_{\mu\nu}$ has been reintroduced using (1) and the equation was reformatted in such a way as to maintain as explicit as possible the symmetrical roles played by the two faces $g_{\mu\nu}$ and $\tilde{g}_{\mu\nu}$ of the Janus field. The contracted form of the DG equation simply is :

$$\sqrt{g}R - \sqrt{\tilde{g}}\tilde{R} = 8\pi G(\sqrt{g}T - \sqrt{\tilde{g}}\tilde{T}) \tag{4}$$

It is well known that "GR can be thought as the unique theory of a massless spin 2 field. So in order to find alternatives to GR, one should break one of the underlying assumptions behind the uniqueness theorem. Breaking Lorentz invariance is probably the most straightforward way..."[36]. DG belongs to this class of theories breaking lorentz invariance and just as the BSLL, Rastall or Rosen theories [35] that have the pre-action requirement ("prior geometry" in the words of Clifford Will) that Riem($\eta_{\mu\nu}$)=0, violations of the strong equivalence principle such as Local lorentz invariance and local position invariance violations are expected[a]. However we anticipate that because of the evolution of an expanding conformal scale factor

[a] In all such theories, even though $\eta_{\mu\nu}$ is a genuine order two tensor field transforming as it should under general coordinate transformations in contrast to a background Minkowski metric $\hat{\eta}_{\mu\nu}$ such as when we write $g_{\mu\nu} = \hat{\eta}_{\mu\nu} + h_{\mu\nu}$, which by definition is invariant since only the transformation of $h_{\mu\nu}$ is supposed to reflect the effect of a general coordinate transformation applied to $g_{\mu\nu}$, $\eta_{\mu\nu}$ actually propagates no degrees of freedom : it is really non dynamical, not in the sense that there is no kinetic (Einstein-Hilbert) term for it in the action, but in the sense that all it's degrees of freedom were frozen a priori before entering the action and need not extremize the action.

in $g_{\mu\nu}$ and the related contraction of the inverse scale factor in $\tilde{g}_{\mu\nu}$ both starting from a finite common value at the Big-Bang (this defines a conformal origin of time t=0), tilde terms should soon after the Big-bang become completely negligible on both sides of our equation which then becomes indistinguishable from GR. All violations of the equivalence principle are then expected to be negligible except near the Big bang or, as we shall see, slightly below the Schwarzschild radius (so both in a regime currently almost impossible to access through direct observations). We will also be able to establish very simply why the instabilities that seem unavoidable due to the ghost interaction between matter and gravity implied by some minus signs in our field equation are for the same reason not menacing the classical stability of an FRW background or a Schwarzschild type solution. The quantum viability is a much more serious issue but we shall argue that DG itself gives us many reasons to believe that gravity is fundamentally not a quantum interaction. To anticipate this discussion, lets say that the semi-classical path (trying to build a viable quantum-classical interaction) is much more natural for DG than GR as the similarity between the gravitational field and other fields is broken by the presence of $\eta^{\mu\nu}$ and the fundamental discrete symmetries at the heart of the theory. See for instance [38] and [81] for an example of specific construction of a quantum-classical interaction. But we would favour yet another approach, one in which the collapse of the wave function is not apparent and progressive (resulting from decoherence) but a real discontinuous and instantaneous phenomenon but still neither menacing causality as we argue in our last section, nor our approximate Bianchi identities.

As for the classical stability of the theory about a Minkowskian background common to the two faces of the Janus field (therefore close to t=0, our Big-bang) we can already argue that:

- Fields (familiar quantum matter and radiation fields) minimally coupled to the two different sides of the Janus field never meet each other from the point of view of the other interactions (EM, weak, strong) so stability issues could only arise in the purely gravitational sector.
- The run away issue [10] [11] is avoided between two masses propagating on $g_{\mu\nu}$ and $\tilde{g}_{\mu\nu}$ respectively, because those just repel each other, anti-gravitationally as in all other versions of DG theories [9][6] rather than one chasing the other ad infinitum.
- The energy of DG gravitational waves almost vanishes about a common Minkowski background (we remind in a forthcoming section that DG has an almost vanishing energy momentum pseudo tensor $t_{\mu\nu} - \tilde{t}_{\mu\nu}$ in this case) avoiding or extremely reducing for instance the instability of positive energy matter fields through the emission of negative energy gravitational waves at or near the Big-Bang.

In particular the first two points are very attractive so we were not surprised discovering that recently the ideas of ghost free dRGT bimetric massive gravity [36] have led to a PN phenomenology identical to our[b].

Also, all such kind of bimetric constructions (our included) seriously question the usual interpretation of the gravitational field as being the metric describing the geometry of space-time itself. There is indeed no reason why any of the two faces $g_{\mu\nu}$ and $\tilde{g}_{\mu\nu}$, which describe a different geometry should be preferred to represent the metric of space-time. At the contrary our non dynamical flat $\eta_{\mu\nu}$ is now the perfect candidate for this role.

We think the theoretical motivations for studying as far as possible a theory such as DG are very strong and three-fold : challenge the idea of background independence, bridge the gap between the discrete and the continuous and challenge the standard understanding of time reversal.

- Challenge the idea of background independence because DG is the straightforward generalization of GR in presence of a background non dynamical metric so either there is no such background and GR is most likely the fundamental theory of gravity or there is one and DG is the most obvious candidate for it.
- Bridge the gap between the discrete and the continuous because we here have both the usual continuous symmetries of GR (in very good approximation) but also a permutation symmetry which is a discrete symmetry between the two faces of the Janus field.
- Challenge the standard understanding of time reversal because as we shall see the two faces of the Janus field are related by a global time reversal symmetry.

The two last points require more clarification and the reader may find enlightening sections in our previous publications (though most of their content is now outdated) however we may summarize the situation as follows:

Basically modern physics incorporates two kinds of laws: continuous and local laws based on continuous symmetries, most of them inherited from classical physics, and discrete and non local rules of the quanta which remain largely as enigmatic today as these were for their first discoverers one century ago. Though there are many ongoing attempts to "unify" the fundamental interactions or to "unify" gravity and quantum mechanics, the unification of the local-continuous with the non-local-discrete laws would be far more fundamental as it would surely come out with a genuine understanding of QM roots. However such unification would certainly require the identification of fundamental discrete symmetry principles underlying the discontinuous physics of the quanta just as continuous and local laws are related to

[b]Indeed the first order differential equation in [32] is exactly the same as our: see e.g eq (3.12) supplemented by (4.10) and for comparison our section devoted to the linearized DG equations. This is because the particular coupling through the mass term between the two dynamical metrics in dRGT eventually constrains them to satisfy a relation Eq (2.4) which for $\alpha = \beta$ [32] becomes very similar to our Eq (1) to first order in the perturbations which then turn out to be opposite (to first order) as Eq (4.10) makes it clear.

continuous symmetries. The intuition at the origin of DG is that the Lorentz group which both naturally involves discrete P (parity) and T (time reversal) symmetries as well as continuous space-time symmetries might be a natural starting point because the structure of this group itself is already a kind of unification between discrete and continuous symmetries. However neither P nor the Anti-Unitary T in the context of QFT seem to imply a new set of dynamical discrete laws. Moreover our investigation in [7] (see also [14] section 3) revealed that following the alternative non- standard option of the Unitary T operator to understand time reversal led to a dead-end at least in flat spacetime: indeed there is an obvious unitary-T symmetric of the usual positive energy field of QFT and this is now a negative energy field creating and annihilating negative energy quanta however this field requires a negative kinetic term in the Lagrangian and accordingly a negative Hamiltonian: the problem then is that the Unitary time reversal alone is not able to link the positive Hamiltonian for the familiar positive energy field to the new negative Hamiltonian for the negative energy field.

However we concluded that it might eventually be possible to understand and rehabilitate negative energies and relate them to normal positive energies through time reversal and then rehabilitate the Unitary time reversal interpretation in a fully consistent way, but only in the context of an extension of GR in which the metric itself would transform non trivially under time reversal. This time reversal not anymore understood as a local symmetry but as a global symmetry implying a privileged time and a privileged origin of time, would jump from one metric to it's T-conjugate. Only such time reversal $x^\mu \Rightarrow x^\mu{}_T$ would retain it's discrete nature inherited from the local Lorentz group but now promoted to a global symmetry because at the contrary to a diffeomorphism, a mere reparametrisation which has no actual physical content as it does not affect the set of inertial coordinates i.e. $\zeta^\alpha(x^\mu) \Rightarrow \zeta^\alpha_T(x^\mu_T) = \zeta^\alpha(x^\mu)$ but rather like an internal symmetry it would really discretely transform one set of inertial coordinates $\zeta^\alpha(x^\mu)$ into another non equivalent one $\zeta^\alpha(x^\mu_T) = \zeta^\alpha_T(x^\mu) \neq \zeta^\alpha_T(x^\mu_T)$ (see [4] section 5), i.e. it would transform a metric into a really distinct one describing a different geometry. The DG solutions that we shall remind in the first sections in the homogeneous-isotropic case impressively confirm that our sought privileged time x^0 is a cosmological conformal time reversing according the global symmetry $x^0 \Rightarrow x^0_T = -x^0$ about a privileged origin of time $x^0 = 0$ and that the two faces of the Janus field are just this time reversal conjugate metrics we have been looking for: in particular the conjugate conformal scale factors are indeed found to satisfy $\tilde{a}(t) = 1/a(t) = a(-t)$ (also see [14] section 6.2). The interpretation of this new global time reversal is also very different from the interpretation of the familiar local time reversal: the later exchanges initial and final states as does the anti-unitary operator of QFT so it means going backward in time whereas our new global time reversal amounts to jump from t to -t and not to go backward in time.

The solutions in the isotropic case then also confirm the reversal of the gravific energy as seen from the conjugate metric i.e any F field is seen as a positive energy field by other F fields (as it produces an attractive potential well in $g_{\mu\nu}$) but as a negative energy field (as it produces a repelling potential hill in $\tilde{g}_{\mu\nu}$) from the point of view of $\tilde{F}$ fields and vice versa. In a sense DG had to reinvent an absolute zero and negative values for the time and mass-energies which only became possible thanks to the pivot metric $\eta_{\mu\nu}$. Eventually the difference with GR is that a coordinate transformation such as time reversal, does not modify the geometry described by the gravitational field in GR whereas in our case it is the geometry described by the couple of conjugate gravitational fields as a whole which is not modified since the two components of the couple are just exchanged.

At last we are aware that we are not yet ready to derive the Planck-Einstein relations from this new framework but in the following we will have to keep in mind what was our initial motivation: understand the origin of the discrete rules of QM from discrete symmetries to not prohibit oneself the explicit introduction of discrete rules and processes any time the development of the theory seems to require them.

The article is organized as follows: in section 2 we remind and complement the results of previous articles as for the homogeneous-isotropic solution and present the full complete test of DG cosmology against the main data: SN, BAO, CMB. In section 3 we comment the local static isotropic asymptotically Minkowskian solutions of the DG equation. In section 4 we discuss the linearized theory about this common Minkowskian background for $g_{\mu\nu}$ and $\tilde{g}_{\mu\nu}$ and the prediction of the theory as for the emission of gravitational waves. In sections 5 and 6, we give up the hypothesis that the two conjugate metrics are asymptotically the same to derive the isotropic static solution again in this more general case and discuss our pseudo Black Hole and new predictions for gravitational waves. In section 7, we investigate the physics of an effective energy exchange through an innovative off-shell mechanism between the two sides of our universe. This exchange was found necessary to avoid static solutions in section 2. In section 8 we start to seriously consider the case of actual static background solutions in some delimited spatial domains and pursue this exploration in section 8 and 9 having in mind a possible explanation of the Pioneer anomaly and renewed understanding of expansion effects. Various other possible predictions are described in section 10. Section 11 explores a plausible MOND like phenomenology of DG. Section 12 discusses all kind of stability issues to conclude that the theory is safe once understood as a semi-classical theory of gravity. Section 13 outlines the DG linear theory of cosmological perturbations. Section 14 analyses a new plausible Dark Matter candidate and mechanisms mimicking the Dark Matter phenomenology within our framework. Section 15 explains how we could get a scale invariant primordial power spectrum. Before the conclusion, section 16 last remarks and outlooks, among other topics, rapidly cover, in turn, the old cosmological problem, the potential issue of closed timelike curves (CTCs) and emphasizes the need for a theory of gravity such as DG which very principles being based on discrete as well as continuous symmetries, for the first time open a natural bridge to quantum mechanics and hopefully offer a royal road toward a genuine unification.

2. The homogeneous and isotropic case

2.1. *Unphysical background solutions*

We found that an homogeneous and isotropic solution is necessarily spatially flat because the two sides of the Janus field about our flat Minkowski background are required to be both homogeneous and isotropic whereas if one of the two metrics is homogeneous and isotropic with non vanishing spatial curvature $k \neq 0$ then the conjugate one is not an homogeneous and isotropic metric.

The conjugate homogeneous and isotropic spatially flat metrics are then assumed to take the form $g_{\mu\nu} = a(t)\eta_{\mu\nu}$ and $\tilde{g}_{\mu\nu} = a^{-1}(t)\eta_{\mu\nu}$. In the coordinate system in which the non dynamical background Minkowski metric $\eta_{\mu\nu}$ reads diag(-1,1,1,1), our metrics then have the conformal form. In the following the time variable t is therefore the conformal time and the Hubble parameters H and $\tilde{H}$ are understood to be conformal Hubble parameters. Then the two Friedman type equations the conformal scale factor should satisfy are:

$$a^2(2\dot{H} + H^2) - \tilde{a}^2(2\dot{\tilde{H}} + \tilde{H}^2) = -6K(a^4 p - \tilde{a}^4\tilde{p}) \tag{5}$$

$$a^2H^2 - \tilde{a}^2\tilde{H}^2 = 2K(a^4\rho - \tilde{a}^4\tilde{\rho}) \tag{6}$$

with $K = \frac{4\pi G}{3}$, but an equivalent couple of equations is:

$$a\ddot{a} - \tilde{a}\ddot{\tilde{a}} = K(a^4(\rho - 3p) - \tilde{a}^4(\tilde{\rho} - 3\tilde{p})) \tag{7}$$

$$\dot{a}^2 - \dot{\tilde{a}}^2 = 2K(a^4\rho - \tilde{a}^4\tilde{\rho}) \tag{8}$$

The time derivative of the second equation leads to:

$$a\ddot{a} + \tilde{a}\ddot{\tilde{a}} = K(a^4\frac{\dot{\rho}}{H} - \tilde{a}^4\frac{\dot{\tilde{\rho}}}{H} + 4\rho a^4 + 4\tilde{\rho}\tilde{a}^4) \tag{9}$$

with $H = \frac{\dot{a}}{a} = -\frac{\dot{\tilde{a}}}{\tilde{a}}$. The energy conservation equations on both sides being:

$$\frac{\dot{\rho}}{H} = -3(\rho + p) \tag{10}$$

$$\frac{\dot{\tilde{\rho}}}{\tilde{H}} = -\frac{\dot{\tilde{\rho}}}{H} = -3(\tilde{\rho} + \tilde{p}) \tag{11}$$

we can replace the corresponding terms in (9),

$$a\ddot{a} - \tilde{a}\ddot{\tilde{a}} = K(a^4(\rho - 3p) - \tilde{a}^4(\tilde{\rho} - 3\tilde{p})) \tag{12}$$

$$a\ddot{a} + \tilde{a}\ddot{\tilde{a}} = K(a^4(\rho - 3p) + \tilde{a}^4(\tilde{\rho} - 3\tilde{p})) \tag{13}$$

then adding and subtracting the two equations we get the new equivalent couple of differential equations:

$$a\ddot{a} = Ka^4(\rho - 3p) \tag{14}$$

$$\tilde{a}\ddot{\tilde{a}} = K\tilde{a}^4(\tilde{\rho} - 3\tilde{p}) \tag{15}$$

which makes clear that the two equations are not compatible with $\tilde{a} = 1/a$ and any usual equation of state except for empty and static universes. For instance in the $a(t) = e^{h(t)}$, $\tilde{a}(t) = e^{-h(t)}$ domain of small h(t), to first order in h, (14)(15) reduce to:

$$\ddot{h} = K(\rho_0 - 3p_0) \geq 0 \tag{16}$$

$$\ddot{h} = -K(\tilde{\rho}_0 - 3\tilde{p}_0) \leq 0 \tag{17}$$

The reason for that incompatibility is that the Bianchi identities are not anymore rigorously satisfied by the lhs of our field equation to make the DG equations functionally dependent as in GR. Of course physical degrees of freedom that were non physical in GR must be released in DG corresponding to these additional equations.

In the homogeneous-isotropic case we can indeed reintroduce the additional time dependent degree of freedom $\zeta(t)$ in the metric: $d\tau^2 = a^2(t)(\zeta^2(t)dt^2 - d\sigma^2)$ which is of course known to be pure Gauge in GR but not in DG as it produces the desynchronization of our clocks with respect to dark side ones. Then the equations involving $\mathscr{C} = \frac{\dot{\zeta}}{\zeta} = -\frac{\dot{\tilde{\zeta}}}{\tilde{\zeta}}$ and ζ are:

$$\frac{a^2}{\zeta}(2\dot{H} + H^2 - 2H\mathscr{C}) - \frac{\tilde{a}^2}{\tilde{\zeta}}(2\dot{\tilde{H}} + \tilde{H}^2 - 2\tilde{H}\tilde{\mathscr{C}}) = -6K(a^4\zeta p - \tilde{a}^4\tilde{\zeta}\tilde{p}) \tag{18}$$

$$\frac{a^2}{\zeta}H^2 - \frac{\tilde{a}^2}{\tilde{\zeta}}\tilde{H}^2 = 2K(a^4\zeta\rho - \tilde{a}^4\tilde{\zeta}\tilde{\rho}) \tag{19}$$

A choice of initial conditions such as $a(t=0) = \tilde{a}(t=0)$, $\zeta(t=0) = \tilde{\zeta}(t=0) = \zeta^{-1}(t=0)$ forces $\rho(t=0) = \tilde{\rho}(t=0)$. So $p(t=0) = \tilde{p}(t=0)$ might also be a natural initial choice (we shall understand later why the initial equality of all conjugate densities is essential to ensure what we believe to be a fundamental time reversal symmetry of the background in DG). Then at t=0 the second equation is trivially satisfied (0=0) while the first is greatly simplified to: $\dot{H}(t=0) = 0$ (remember that by construction we have $H = -\tilde{H}$, $\mathscr{C} = -\tilde{\mathscr{C}}$, $H^2 = \tilde{H}^2$ so $H\mathscr{C} = \tilde{H}\tilde{\mathscr{C}}$ at anytime) so that remarkably at t=0 both equations neither constrain $H(t=0)$ nor $\mathscr{C}(t=0)$ so we may impose $\mathscr{C}(t=0) = 0$ just as an additional initial condition. Noticeably the choice $\zeta \propto a^2$ i.e. $\mathscr{C} = 2H$ allows a pair of empty universes to evolve exponentially for ever. For non empty universes we can check whether there exists solutions of our two equations compatible with the equations of motion satisfied by our sources. Following the same steps as before lets get the equivalent sets of equations:

$$a\frac{\ddot{a}}{\zeta} + \tilde{a}\frac{\ddot{\tilde{a}}}{\tilde{\zeta}} - H\frac{\mathscr{C}}{2}\left(\frac{a^2}{\zeta} + \frac{\tilde{a}^2}{\tilde{\zeta}}\right) = K(a^4(\rho - 3p)\zeta + \tilde{a}^4(\tilde{\rho} - 3\tilde{p})\tilde{\zeta} + \frac{\mathscr{C}}{H}(a^4\zeta\rho + \tilde{a}^4\tilde{\zeta}\tilde{\rho})) \tag{20}$$

$$a\frac{\ddot{a}}{\zeta} - \tilde{a}\frac{\ddot{\tilde{a}}}{\tilde{\zeta}} - H\mathscr{C}\left(\frac{a^2}{\zeta} - \frac{\tilde{a}^2}{\tilde{\zeta}}\right) = K(a^4(\rho - 3p)\zeta - \tilde{a}^4(\tilde{\rho} - 3\tilde{p})\tilde{\zeta}) \tag{21}$$

and then:

$$2a\frac{\ddot{a}}{\zeta}+H\mathscr{C}(-\frac{3a^2}{2\zeta}+\frac{\tilde{a}^2}{2\tilde{\zeta}})=K(2a^4(\rho-3p)\zeta+\frac{\mathscr{C}}{H}(a^4\zeta\rho+\tilde{a}^4\tilde{\zeta}\tilde{\rho})) \quad (22)$$

$$2\tilde{a}\frac{\ddot{\tilde{a}}}{\tilde{\zeta}}+H\mathscr{C}(\frac{a^2}{2\zeta}-\frac{3\tilde{a}^2}{2\tilde{\zeta}})=K(2\tilde{a}^4(\tilde{\rho}-3\tilde{p})\tilde{\zeta}+\frac{\mathscr{C}}{H}(a^4\zeta\rho+\tilde{a}^4\tilde{\zeta}\tilde{\rho})) \quad (23)$$

Numerically solving these equations in a simple case (assuming no pressure) we now realize that in general one cannot impose two initial conditions independently on $\mathscr{C}(t=0)$ and $H(t=0)$ otherwise our cosmology is blocked. Thus the initial conditions are $a(t=0)=\tilde{a}(t=0)$, $\zeta(t=0)=\tilde{\zeta}(t=0)=\zeta^{-1}(t=0)$, $\rho(t=0)=\tilde{\rho}(t=0)$ and a value for $H(t=0)$ which in turn determines $\mathscr{C}(t=0)$. We nevertheless would like to stress that at the level of equations 18 and 19 the choice of $\mathscr{C}(t=0)=0$ just as any other choice was perfectly allowed as an initial condition whatever $H(t=0)$. It's now because of the matter-radiation equations that need to be fulfilled simultaneously, that this is not anymore the case.

Given that the solutions obtained numerically are not physically very appealing (densities are always monotonous function of time as far as we could investigate them with Mathematica) we feel free to still make more physically interesting choices such as the frozen ζ version coming with the initial condition $\dot{\zeta}(t=0)=\mathscr{C}(t=0)=0$ that implies the Lorentz invariance of the two conjugate cosmological backgrounds (under the same transformation that leave invariant the global non dynamical Minkowski metric) and the same speed of GWs and light on these backgrounds.

But another physically interesting choice is $\zeta\propto\frac{1}{a^4}$ i.e. $\mathscr{C}=-4H$ (so another initial condition $\mathscr{C}(t=0)=-4H(t=0)$) that could be imposed by the requirement that cosmological constant source terms (possibly infinite vacuum energy terms from both sides) should cancel out at anytime. Indeed solving this way the old cosmological constant problem, the equations become:

$$a^6(2\dot{H}+9H^2)-\tilde{a}^6(2\dot{\tilde{H}}+9\tilde{H}^2)=-6K(p-\tilde{p}) \quad (24)$$

$$a^6H^2-\tilde{a}^6\tilde{H}^2=2K(\rho-\tilde{\rho}) \quad (25)$$

All along the study of the frozen ζ version we will keep an eye on the $\zeta\propto\frac{1}{a^4}$ variant of DG, and highlight the difference in their predictions. But both need the mechanism presented in the next section to unblock the cosmology by releasing an additional dof. Indeed, at t=0, equations 22 and 23 imply (with the simplifying assumption that pressures vanish):

$$\ddot{a}(t=0)=\ddot{\tilde{a}}(t=0)=H^2=\frac{H\mathscr{C}}{2}+K\rho(1+\frac{\mathscr{C}}{H}) \quad (26)$$

thus $H(t=0)=\sqrt{K\rho}$ for $\dot{\zeta}(t=0)=\mathscr{C}(t=0)=0$ (but this does not prevent $\zeta(t)$ to evolve at $t>0$), and no solution at all ($H(t=0)=\sqrt{-K\rho}$, $K>0$) for $\mathscr{C}(t=0)=-4H(t=0)$. We now stick to the frozen $\zeta(t)$ version (a priori $H(t=0)\neq\sqrt{K\rho}$ but the evolution of the scale factor will be unlocked by an extra dof to be introduced in the next section). We would like to emphasize that we are not, in any way, arbitrarily removing physical dofs: at the level of the action all elements of the metrics were varied to get the cosmological equations. We are just choosing a set of perfectly allowed initial conditions $\mathscr{C}(t=0)$=0 and an unconstrained $H(t=0)$ at will. But this in turn requires a re-investigation and revision of the conservation laws of the source matter and radiation fields on the rhs of the equations to unlock cosmology (at least when $H(t=0) \neq \sqrt{K\rho}$).

Actually our initial condition choice for the birth of a couple of conjugate universes, the imposed Lorentz invariance of the initial background metric is the requirement that not only $\mathscr{C}(t=0)$=0 but also all its time derivatives vanish at t=0. This in turn may insure that the background remains Lorentz invariant at any time if $\mathscr{C}$ is analytical so that it's Taylor series always identify with the function (this suggests that may be it will be interesting to consider and explore in the future an extension of DG in which time is a complex number) but it remains a condition on the initial state only. The existence of a genuine initial state allowing such initial conditions to be imposed gives the conformal scale factor a very special status in DG, with no equivalent neither in GR nor other alternative metric theories of gravity as far as we know. This will make possible postulating discrete transitions for the scale factor as we shall do soon.

It remains that the Bianchi identities remain an extremely good approximation as far as $\tilde{a} >> a$ or $a >> \tilde{a}$ and even when the scale factors are not much different, linear Bianchi identities are valid at the linear level so that we can still apply all GR reasoning and methods to identify the main Gravitational Waves propagating degrees of freedom. To get the linearized DG theory, linearization which as usual neglects the effects of expansion by assuming an asymptotically Minkowskian metric, we would write $g_{\mu\nu} = C(\eta_{\mu\nu} + h_{\mu\nu})$ and $\tilde{g}_{\mu\nu} = C^{-1}(\eta_{\mu\nu} + \tilde{h}_{\mu\nu})$ with $\tilde{h}_{\mu\nu} = -h_{\mu\nu}$ neglecting non linear terms and with a constant C factor because the asymptotic metrics need not be the same on both sides and it is straightforward to verify that the linear Bianchi identities are verified (verified independently for each of the two terms that we have on the lhs of our DG equations) as well as the Gauge invariance of our gravitational field Lagrangian under any weak field transformation $h_{\mu\nu} \rightarrow h'_{\mu\nu} = h_{\mu\nu} - \frac{\partial \epsilon_\mu}{\partial x^\nu} - \frac{\partial \epsilon_\nu}{\partial x^\mu}$, $\tilde{h}_{\mu\nu} \rightarrow \tilde{h}'_{\mu\nu} = \tilde{h}_{\mu\nu} - \frac{\partial \epsilon_\mu}{\partial x^\nu} - \frac{\partial \epsilon_\nu}{\partial x^\mu}$ just as in General Relativity. So it is only at the non linear level that the Bianchi identities are not anymore exactly fulfilled and that DG loses the GR active diffeomorphism invariance [c] [d] : in the sense that our field equations are not invariant under the transformations of $g_{\mu\nu}$ alone but under the combined transformations of $g_{\mu\nu}$ and $\eta_{\mu\nu}$.

2.2. *Effective energy exchange by varying gravitational couplings*

As they stand the DG equivalent (7) of GR Friedman equations are not viable. Our previous discussion about the Bianchi identities not being rigorously satisfied outlined the origin of the problem: because our choice of initial conditions freezes $\zeta(t)$, we have more cosmological equations (2) than degrees of freedom (1 scale factor). The only possible solution thus seems to be the introduction of an additional degree of freedom without any new equation i.e. a non dynamical scalar, in the sense that it should not extremize the action avoiding thereby any additional field equation for it. Inspired by an original idea by Prigogin (see for instance [49] and multi-references therein) in an earlier version of this work we allowed the adiabatic creation or annihilation of particles on either side. Our conservation equations then got modified[e]:

$$\dot{\rho} = (\Gamma - 3H)(\rho + p) \tag{27}$$

$$\dot{\tilde{\rho}} = (\tilde{\Gamma} - 3\tilde{H})(\tilde{\rho} + \tilde{p}) \tag{28}$$

The next assumption was then to relate the creation rates through $\tilde{\Gamma} = -\Gamma$ (just as $\tilde{H} = -H$) in such a way that in principle no actual creation or annihilation of particles would be needed but merely a particle transfer from one metric to the conjugate [f] with baryonic number conservation but with a re-scaling of those

[c]In the words of C. Rovelli :"Active diff invariance should not be confused with passive diff invariance, or invariance under change of coordinates. GR can be formulated in a coordinate free manner, where there are no coordinates, and no changes of coordinates. In this formulation, there field equations are still invariant under active diffs. Passive diff invariance is a property of a formulation of a dynamical theory, while active diff invariance is a property of the dynamical theory itself. A field theory is formulated in manner invariant under passive diffs (or change of coordinates), if we can change the coordinates of the manifold, re-express all the geometric quantities (dynamical and non-dynamical) in the new coordinates, and the form of the equations of motion does not change. A theory is invariant under active diffs, when a smooth displacement of the dynamical fields (the dynamical fields alone) over the manifold, sends solutions of the equations of motion into solutions of the equations of motion. Distinguishing a truly dynamical field, namely a field with independent degrees of freedom, from a non dynamical field disguised as dynamical (such as a metric field g with the equations of motion Riemann[g]=0) might require a detailed analysis (for instance, hamiltonian) of the theory." [83]

[d]Another example of theory violating active diffeomorphism invariance is for instance unimodular gravity [43][44]

[e]The equations are as valid in conformal time as in standard time. The conformal time Γ and H here are related to the standard time t' for our side metric Γ' and H' according $\Gamma = a\Gamma'$ and $H = aH'$. The standard time being t" for the conjugate metric we also have $\tilde{\Gamma} = \tilde{a}\tilde{\Gamma}''$ and $\tilde{H} = \tilde{a}\tilde{H}''$

[f]Also notice that contrary to [49] in our case since the Bianchi identities can only approximately be verified on the left hand side, the right hand side can involve the energy-momentum conservation violating tensors (very weak violation when the ratio of the scale factors is very large as we shall see).

particle energies since for vanishing pressures the previous equations imply $\frac{\dot{\rho}}{\rho} = -\frac{\dot{\tilde{\rho}}}{\tilde{\rho}}$, rather than $\dot{\rho} = -\dot{\tilde{\rho}}$. This rescaling then makes the interpretation of $\tilde{\Gamma} = -\Gamma$ not so natural in terms of matter and radiation exchange. We now have opted for a better alternative avoiding the introduction of new actions or complicated mechanisms behind $\tilde{\Gamma} = -\Gamma$: just let the offshell variation in time of the two sides adimensional (being divided by their common initial value G_0 at t=0) gravitational "constants" $G(t)$ and $\tilde{G}(t)$ inside the matter-radiation actions but also then included in the redefined densities and pressures, assuming them to be related by $\tilde{G}(t) = 1/G(t)$ from which follows slightly modified non conservation equations :

$$\dot{\rho} = \Gamma\rho - 3H(\rho + p) \tag{29}$$

$$\dot{\tilde{\rho}} = \tilde{\Gamma}\tilde{\rho} - 3\tilde{H}(\tilde{\rho} + \tilde{p}) \tag{30}$$

with $\Gamma = \frac{\dot{G}}{G} = -\tilde{\Gamma} = -\frac{\dot{\tilde{G}}}{\tilde{G}}$. Such equations can be derived following the same method used to derive the covariant energy-momentum tensor conservation in GR from the fact that the matter actions are scalars except that now we have an extra contribution coming from the offshell gravitational "constants" entering the actions.

With these equations and our still unmodified DG equations, we were led to almost the same phenomenology as the one following from the first postulated matter radiation exchange : the slight difference is only in the influence of the matter and radiation equations of state.

[g]

Now that we have our additional offshell degree of freedom $\Gamma(t)$ non trivial solutions are expected. Replacing again in the differential equations and again adding and subtracting them we alternatively get:

$$a\ddot{a} = K(a^4(\rho - 3p) + \frac{1}{2}(C_r + \tilde{C}_r)) \tag{31}$$

$$\tilde{a}\ddot{\tilde{a}} = K(\tilde{a}^4(\tilde{\rho} - 3\tilde{p}) + \frac{1}{2}(C_r + \tilde{C}_r)) \tag{32}$$

including the creation/annihilation terms $C_r = a^4\frac{\Gamma}{H}\rho$, $\tilde{C}_r = \tilde{a}^4\frac{\Gamma}{H}\tilde{\rho}$. with now the still constant $K = \frac{4\pi G_0}{3}$. Most of the time, in the rest of the article we shall omit the subscript 0 for G_0 in the field equations.

When our side density source terms dominate ($a^4 d >> \tilde{a}^4\tilde{d}$) where d (resp $\tilde{d}$) is any linear combination of densities ρ and p (resp $\tilde{\rho}$ and $\tilde{p}$) alone, we just need

[g]Actually a combined variation of other fundamental constants producing a variation of densities and pressures would also do the job, for instance for the Planck constants, $\Gamma = \frac{\dot{h}}{h} = -\frac{\dot{\tilde{h}}}{\tilde{h}} = -\tilde{\Gamma}$ changes the energies of free massless or massive particles at the same rate (any rest energy m_0 can presumably be written as h ν_0). But then the non gravitational sector is going to be affected as well since this should also affect the fine structure constant α. If we prefer to keep α constant a variation of the electric charge could be also implied. But it's probably more interesting to let α vary as we shall soon argue.

$\frac{\Gamma}{H} << 1$ to recover from the first of these equations, the same evolution law of the scale factors we had in GR. The good new is that now the second equation can be compatible with this solution provided the C_r term is dominant in the second equation : $\frac{\Gamma}{H} >> \frac{\tilde{a}^4\tilde{d}}{a^4 d}$. Then for instance in matter dominated eras on both sides, the equations simplify a bit:

$$a\ddot{a} \approx K a^4 \rho \tag{33}$$

$$\tilde{a}\ddot{\tilde{a}} \approx K \frac{a^4\rho}{2}\frac{\Gamma}{H} \tag{34}$$

from which we get the required evolution of Γ:

$$\Gamma \approx 2H\frac{\tilde{a}\ddot{\tilde{a}}}{a\ddot{a}} = \frac{2H}{a^4}\left(\frac{1-\frac{\dot{H}}{H^2}}{1+\frac{\dot{H}}{H^2}}\right) \tag{35}$$

For a power law $a(t) \propto t^\alpha$ of the conformal scale factor,

$$\Gamma \approx \frac{2\alpha}{a^{4+1/\alpha}}\left(\frac{\alpha+1}{\alpha-1}\right) \tag{36}$$

is positive (energy transfer from the conjugate to our side) for $\alpha > 1$ or $-1 < \alpha < 0$ and negative (energy transfer from our to the conjugate side) otherwise. α positive (resp negative) translates to a decelerating (resp accelerating) universe in standard time t'. Hence in a cold matter dominated era, $\alpha = 2$ (the solutions are presented in greater detail in the next subsection) implies that energy is transferred from the conjugate to our side.

When the conjugate scale factor dominates, roles are exchanged so:

$$\tilde{a}\ddot{\tilde{a}} \approx K \tilde{a}^4 \tilde{\rho} \tag{37}$$

$$a\ddot{a} \approx K \frac{\tilde{a}^4\tilde{\rho}}{2}\frac{\Gamma}{H} \tag{38}$$

then,

$$\Gamma \approx 2H\frac{a\ddot{a}}{\tilde{a}\ddot{\tilde{a}}} = \frac{2H}{\tilde{a}^4}\left(\frac{1+\frac{\dot{H}}{H^2}}{1-\frac{\dot{H}}{H^2}}\right) \tag{39}$$

For a power law $a(t) \propto t^\alpha$ of the conformal scale factor, the sign of

$$\Gamma \approx \frac{2\alpha}{a^{-4+1/\alpha}}\left(\frac{\alpha-1}{\alpha+1}\right) \tag{40}$$

behaves as before and now taking $\alpha = -2$ for an accelerating universe (see next subsection), energy is still transferred from the conjugate to our side.

We see that DG equations can be solved for physically acceptable solutions, i.e. a non static scale factor evolution. This conclusion is actually valid at all times as we could check by numerically integrating our differential equations. We did this assuming for instance $\tilde{p} = p = 0$ (this is just an example, the exercise would work as well for any equations of state) and then $\tilde{\rho} = \rho^{-1}$. Those equations of state of course can't be valid at anytime but the important point is that the same equations of state can be valid on both sides near the origin of time when we have the equality of conjugate densities there. The purpose of this example is actually just to understand the effect of Γ near the origin of time. The system of (necessarily) first order equations integrated thanks to Geogebra NresolEquadiff is:

$$\dot{a} = b \tag{41}$$

$$\dot{b} = \frac{a}{a^2 + \frac{1}{a^2}}\left(\frac{2b^2}{a^4} + K(a^4\rho - \frac{1}{a^4\rho})\right) \tag{42}$$

$$\dot{\rho} = \rho\frac{b}{a}(\Gamma/H - 3) \tag{43}$$

with $\Gamma/H = \frac{\frac{\dot{b}}{a}(a^2 - \frac{1}{a^2}) + 2\frac{b^2}{a^4}}{K(a^4\rho + \frac{1}{a^4\rho})} - 1$. The two first order equations of this system are equivalent to the second order equation $a\ddot{a} - \tilde{a}\ddot{\tilde{a}} = K(a^4\rho - \tilde{a}^4\tilde{\rho})$ while $\frac{\Gamma}{H}$ is deduced from the other second order equation $a\ddot{a} + \tilde{a}\ddot{\tilde{a}} = K(a^4\rho + \tilde{a}^4\tilde{\rho})(1 + \frac{\Gamma}{H})$ still neglecting pressure terms. The resulting functions $a(t)$ and $\rho(t)$ of Figure 1 show that the density increases very sharply near t=0 because of Γ producing like an effective energy density exchange from the dark side to our side while the scale factor remains almost constant. The density reaches a maximum for $\Gamma/H = 3$ then decreases as a^{-3} as expected for pressureless matter when matter exchange becomes negligible. This occurs as soon as our side scale factor has started to dominate over $\tilde{a} = 1/a$, and then this scale factor evolves as t^2 corresponding to $t'^{2/3}$ in standard comoving time coordinate.

The Hubble rate at t=0 is the initial condition that determines the fraction of conformal time the universe spent in the density increasing regime vs the density decreasing regime. For a large enough initial Hubble rate the Γ dominated first stage duration is extremely short compared to the age of the universe since t=0. But this remains unknown as, of course, going backward in time we don't know at which redshift the two conjugate scale factors meet each other.

Again, it is important to understand that the only way to "reconcile" our two cosmological equations was to introduce an additional degree of freedom, which here is our scalar function Γ which must remain offshell (should not extremize the action) otherwise we would also have an additional equation for it, hence still more equations than degrees of freedom. In a sense it appears that the non dynamical $\eta_{\mu\nu}$ and the frozen ζ condition (initial condition choice) require the introduction of another non dynamical object : the scalar G(t).

It is straightforward to check that the $\zeta \propto \frac{1}{a^4}$ variant of DG (see equations 24 25) can be unlocked by the exact same mechanism producing as well a huge effective density exchange between the two sides near t=0. But of course the mechanism should only affect (be introduced) in the normal matter conjugate terms in the actions and not the cosmological constant terms otherwise the old cosmological constant problem would be back. Though this is apparently not a natural condition for a vacuum energy cosmological constant term we shall still keep an eye on the

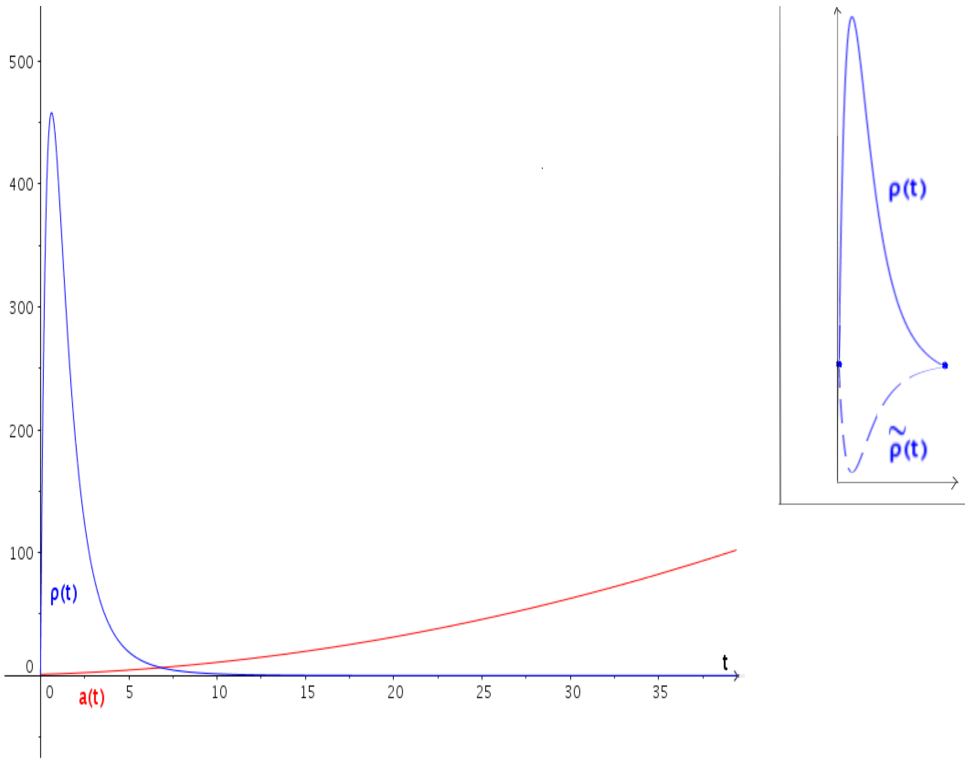

Fig. 1. a(t) and $\rho(t)$ with arbitrary units when including the effect of the effective transfer rate Γ to restore the consistency of Friedmann and conservation equations. Notice that in this figure, a(0)=1 ! The dashed line for $\tilde{\rho}(t) = 1/\rho(t)$ in the upper right corner is only indicative as it was drawn by hand

$\zeta \propto \frac{1}{a^4}$ variant in the forthcoming sections. It remains that far enough from t=0, Γ is again negligible and the familiar dilution and contraction laws are recovered. All tilde terms in our equations will then remain negligible until the conjugate densities get closer to the crossing point.

2.3. *Cosmology*

We are then ready to investigate our cosmological solutions with the insurance that our introduced effective exchange mechanism makes these actual physical solutions. This subsection reviews and provides a more in depth analysis of results already obtained in [14][15].

2.3.1. *Reproducing GR cosmology*

The expansion of our side implies that the dark side of the universe is in contraction. Provided dark side terms and the exchange terms can be neglected which is certainly an excellent approximation far from t=0, our cosmological equations reduce to equations known to be also valid within GR. For this reason it is straightforward for DG to reproduce the same scale factor expansion evolution as obtained within the standard LCDM Model at least up to the redshift of the LCDM Lambda dominated era when something new must have started to drive the evolution in case we want to avoid a cosmological constant term. The evolution of our side scale factor before the transition to the accelerated regime is depicted in blue on the top of Figure 2 as a function of the conformal time t and the corresponding evolution laws as a function of standard time t' are also given in the radiative and cold era. Notice however the new behaviour about t=0 meaning that the Big-Bang singularity is avoided.

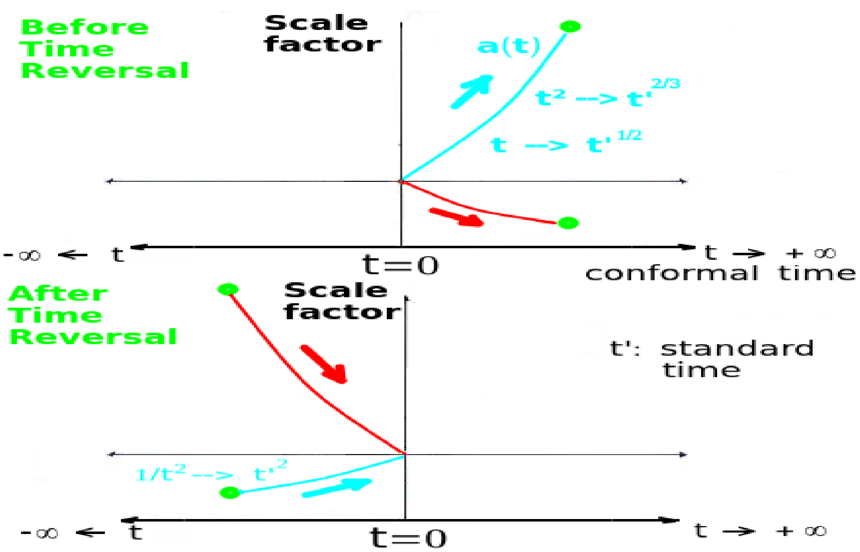

Fig. 2. Evolution laws and time reversal of the conjugate universes, our side in blue

2.3.2. *Continuous evolution and discontinuous permutation*

A discontinuous transition is a natural possibility within a theory involving truly dynamical discrete symmetries as is our permutation symmetry in DG. The basic idea is that some of our beloved differential equations might only be valid piecewise, only valid in the bulk of space-time domains at the frontier of which new discrete rules apply implying genuine field discontinuities. Here this will be the case for the scale factor. Of course a discontinuous process can't be consistent with the continuous process predicted by a differential equation but here the two kind of processes have their own domain of validity (the bulk vs the frontier) which avoids any conflicting predictions. However we would prefer the discontinuous process not to occur arbitrarily but to be governed by the same discrete symmetries readily readable from the equations of motion.

We postulated that a transition occurred billion years ago as a genuine permutation of the conjugate scale factors, understood to be a discrete transition in time modifying all terms explicitly depending on $a(t)$ but not the densities and pressures themselves in our cosmological equations: in other words, the equations of free fall apply at any time except the time of the discrete transition.

Let's be more specific. The equations of free fall for the perfect fluids on both sides of course apply as usual before and after the transition and for instance on our side in the cold era dominated by non relativistic matter with negligible pressure, we have $\frac{d}{dt}(\rho a^3) = 0$. Such conservation equation is valid just because it follows from our action for the matter fields on our side. But here we not only have the usual invariance of our action under continuous space-time symmetries from which we can derive the corresponding field conservation equations closely related to the continuous field equations of motion valid in the bulk of a space-time domain. We also have the invariance of the action under a permutation which is a discrete symmetry. To continuous symmetries can be associated continuous evolution, interactions and conservation equations of the fields thanks to variational methods. Such methods are of course not available to derive discontinuous processes from discrete symmetries so we postulate and take it for granted that our new permutation symmetry also allows a new kind of process to take place : the actual permutation of the conjugate a and $\tilde{a}$ while density and pressure terms remain unchanged. Because such process is not at all related to the continuous symmetries that generate the continuous field equation there is indeed no reason why the discrete version $(\rho a^3)_{before} = (\rho a^3)_{after}$ of a conservation equation such as $\frac{d}{dt}(\rho a^3) = 0$ should be satisfied by this particular process. The symmetry principles and their domain of validity are the more fundamental so we should not be disturbed by a process which violates the conservation of energy since this process is discontinuous, only valid at the frontier of a space-time domain and related to a new discrete symmetry for which we have no equivalent of the Noether theorem. Here the valid rule when the permutation of the scale factors occurs is rather $\rho_{before} = \rho_{after}$ and the same for the pressure densities.

This permutation (at the green point depicted on figure 2) could produce the subsequent recent acceleration of the universe. This was already understood in previous articles [14] and [15] assuming our side source terms such as $a^4(\rho - 3p)$ have been dominant and therefore have driven the evolution up to the transition to acceleration. Specifically, just before the transition we have for instance: $a^4(\rho - 3p) >> \tilde{a}^4(\tilde{\rho} - 3\tilde{p})$ just because $a(t) >> \tilde{a}(t)$ and $\rho - 3p \approx \tilde{\rho} - 3\tilde{p}$ resulting in the usual (as in GR) expansion laws whereas just after the transition, $a^4(\rho - 3p) << \tilde{a}^4(\tilde{\rho} - 3\tilde{p})$ because now $a(t) << \tilde{a}(t)$ and $\rho - 3p \approx \tilde{\rho} - 3\tilde{p}$ resulting in the dark side source term now driving the evolution, producing a constant acceleration of our side scale factor in standard time coordinate t' following the transition redshift : $a(t') \propto t'^2$. In fact the reason why the densities do not change at the transition is that actually this transition is understood to be triggered by the crossing of conjugate densities ($\rho = \tilde{\rho}$ and $p = \tilde{p}$). Indeed, in general our cosmological equations are actually invariant under the combined permutations of densities and scale factors rather than permutation of scale factors alone so we might have expected from this symmetry that the allowed discontinuous process should exchange scale factors as well as densities simultaneously. However when the densities are equal our equations become invariant under the exchange of scale factors alone so the discontinuous process does not need to actually exchange the densities at this time but only the scale factors. Moreover we then have the bonus that the equality of densities is a perfect triggering condition for the transition to occur and we already knew from the previous section analyses (upper right corner of Figure 1) that the crossing of densities is anyway expected.

2.3.3. *Global time reversal and permutation symmetry*

The evolution of the scale factor is largely determined by initial conditions at t=0. The parameters are the initial densities ρ_o, p_o, $\tilde{\rho}_o$, $\tilde{p}_o$ and initial expanding rate H_o (not to be confused with the usual present standard time t' Hubble rate H'_0). Considering a scenario with equal initial densities on both sides one needs a non vanishing H_o to get non static solutions which then turn out to satisfy the fundamental relation:

$$\tilde{a}(t) = \frac{1}{a(t)} = a(-t) \tag{44}$$

For this reason, already in our previous publications we could interpret our permutation symmetry as a global time reversal symmetry about privileged origin of conformal time t=0. But from such initial conditions (equal initial densities) it would erroneously appear that the densities (decreasing on our expanding side while increasing on the contracting dark side) will never have the opportunity to cross again. This is not exact however as soon as we acknowledge the crucial role of the significant continuous matter-radiation exchange near the origin of time. Indeed, thanks to matter-radiation exchange we can now have equal conjugate densities at the origin of time that will again be equal in the future according our previous subsection results and as can be readily seen from Figure 1.

Without such exchange mechanism, we know that our differential equations have no solutions except the trivial static ones but just out of curiosity we may consider the fictitious theory of conjugate "scalar-eta" fields $\phi\eta_{\mu\nu}$ and $\phi^{-1}\eta_{\mu\nu}$. This scalar field $\phi(t) = a^2(t)$ in the homogeneous case now only needs to satisfy the single differential equation 7. The value of considering such fictitious scalar theory is that it does not require us to postulate any exchange mechanism to get realistic solutions for the scale factor. For such theory, to get benefit from our scale factors permutation postulated process (A) we would need to break the initial equality between densities in such a way that the densities could again cross each other at a time different from t=0. Then however, we would realize that for $a(t) = e^{h(t)}$, h(t) is not anymore an odd function meaning that the condition Eq 44 for interpreting the permutation symmetry as a global time reversal would be broken. The only thing we would need to restore Eq 44 is to postulate another discrete process (B), again a density exchange process occurring at t=0 but now a discrete one. This is illustrated in fig 3 where h(t) is plotted with (plain line) and without (dotted line) assuming such exchange. The value of this fictitious scalar theory example is to make us realize that fortunately, thanks to the continuous matter exchange mechanism of our actual theory we get Eq 44 for free i.e. without any need to postulate an additional discrete process such as (B) at t=0.

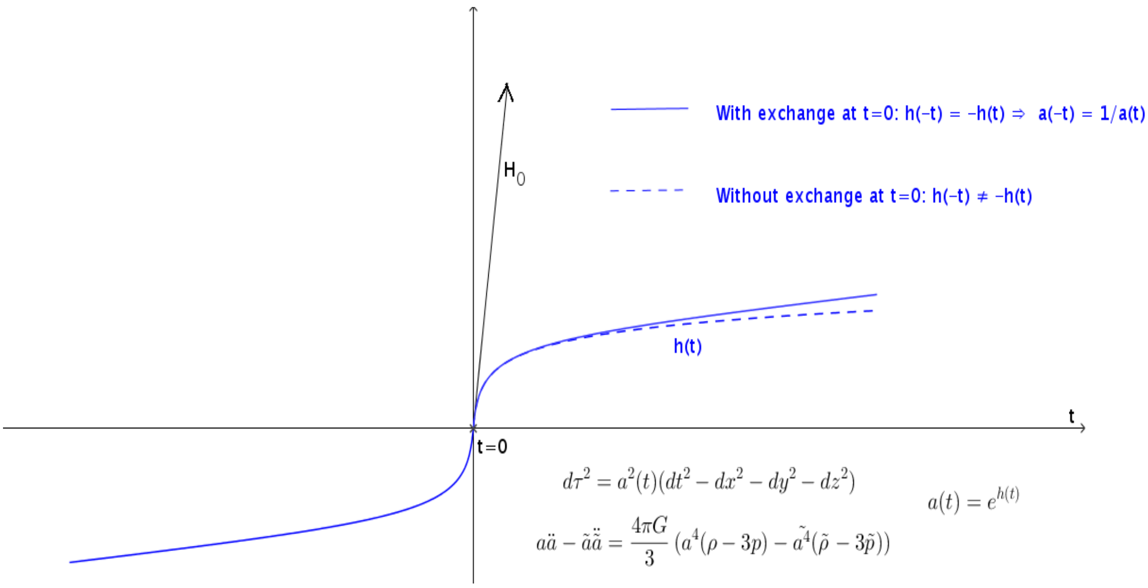

Fig. 3. h(t) with or without discrete exchange of densities at t=0 in a scalar-eta fictitious theory

Anyway, whether continuous or discontinuous, densities exchange processes result in the inversion of densities evolution laws i.e from decreasing to increasing or vice versa, so that the evolution of both densities and scale factors are cyclic as illustrated in fig 4. This also insures the stability of our homogeneous solutions in the sense that these remain bounded and confirms that we completely avoid any singularity issue.

By the way having equal initial densities is also ideal to have equal amounts of matter and anti-matter at the origin of time, but then, following the separation of the two sides, a small excess of matter on our side corresponding to the same exact small excess of anti-matter on the conjugate side. The small excess on our side would then presumably be the origin of the baryonic asymmetry of our universe after almost complete matter anti-matter annihilation.

Once our permutation symmetry is successfully reinterpreted as being associated with a time reversal symmetry, for the scale factors to exchange their respective

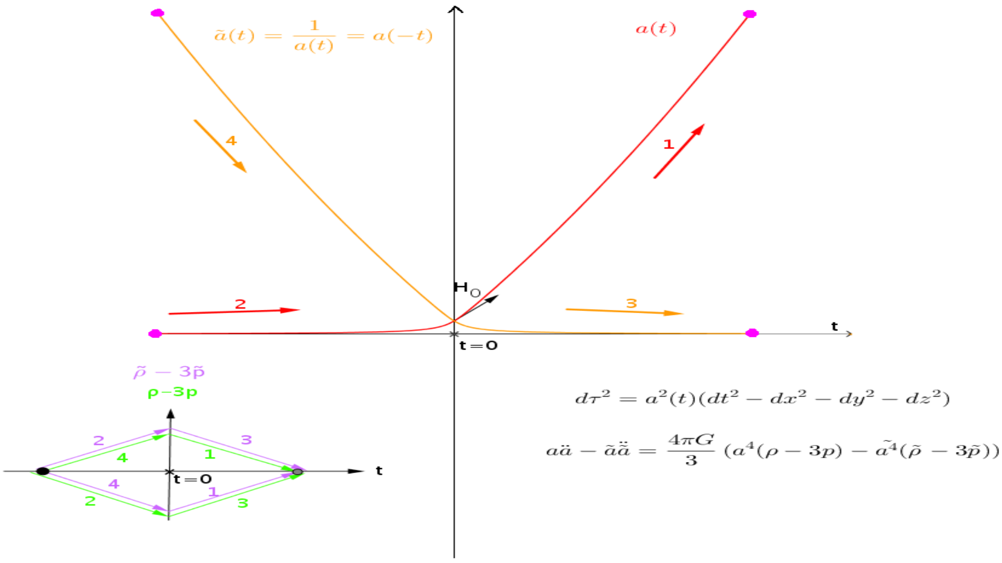

Fig. 4. Scale factors and densities evolution for a fictitious "scalar-eta" theory

values at the equality of densities, we just need to jump from t to -t as illustrated in fig 2 and 4. A mere permutation symmetry would also exchange the scale factors time derivatives producing an inversion of the arrow of time and therefore Hubble rates i.e. a transition from expansion to contraction on our side. So our time reversal symmetry is actually only a permutation of the scale factors while the Hubble rates and densities remain the same (symmetry also satisfied by our differential equations) resulting in our side still being expanding as promised following the transition redshift.

2.3.4. *Discontinuities and consistency checks*

To gain insight into the meaning of field discontinuities, let us first investigate the possibilities offered to us within GR. Assume space-time can be divided into two domains D_- and D_+ separated by a constant conformal time hypersurface t=T. In the domain $D_- =]-\infty, T_-[$ the laws of GR apply just as they also apply in $D_+ =]T_+, +\infty[$. The question is whether we may consider a non trivial i.e. non continuous relation linking the D_+ matter-radiation and gravitational fields and derivatives in the T_+ limit to D_- matter-radiation and gravitational fields and derivatives in the T_- limit. Of course the problem is severely constrained by equations of motion that must be satisfied in both domains however the solutions are not only determined by the equations but also by integration constants that we may chose differently in the two domains by imposing different asymptotic conditions at infinity, and these in turn could imply discontinuities i.e. non matching D_+ and D_- solutions in the limit t=T.

The homogeneous case with negligible pressures is the simplest one to start with. In D_+ and D_- the set of independent equations are the first Friedmann equation and the conservation equation of matter fields.

$$H_+^2 = \frac{8\pi G}{3}\rho_+ a_+^2 \qquad \frac{\dot{\rho}_+}{\rho_+} = -3H_+ \tag{45}$$

$$H_-^2 = \frac{8\pi G}{3}\rho_- a_-^2 \qquad \frac{\dot{\rho}_-}{\rho_-} = -3H_- \tag{46}$$

in which H_+ and H_- are still conformal Hubble parameters. In a conservative approach (also motivated by the kind of discontinuity we are interested in within DG), we are wondering whether a discontinuity of the scale factor implying $a_+(T_+) = Ca_-(T_-)$ and implying a mere renormalization by a constant C of the total gravitational field from one domain to the other, while matter and radiation densities (and their derivatives) would be continuous is a possibility within GR. This of course also implies the continuity of the Hubble parameters : $H_+(T_+) = H_-(T_-)$.

The conservation equations do not forbid $\rho_+(T_+) = \frac{c_+}{a_+^3(T_+)} = \frac{c_-}{a_-^3(T_-)} = \rho_-(T_-)$ as the discontinuity of the scale factor $(a_+(T+) \neq a_-(T_-))$ can be compensated by different integration constants $c_+ \neq c_-$ to maintain the continuity of the density $\rho_+(T_+) = \rho_-(T_-)$. However then the Friedmann equations in the two domains obviously can't be consistent ! Such kind of discontinuity is therefore forbidden within GR but what about DG ? Again we know that thanks to various integration constants a discontinuity (by a renormalization constant) of the scale factor leaving the densities and Hubble rates continuous just as in the above GR case is not an issue as far as the matter and radiation conservation equations are concerned. Now the corresponding first Friedmann-DG equations are:

$$a_+^2 H_+^2 - \tilde{a}_+^2 \tilde{H}_+^2 = \frac{8\pi G}{3}(\rho_+ a_+^4 - \tilde{\rho}_+ \tilde{a}_+^4) \tag{47}$$

$$a_-^2 H_-^2 - \tilde{a}_-^2 \tilde{H}_-^2 = \frac{8\pi G}{3}(\rho_- a_-^4 - \tilde{\rho}_- \tilde{a}_-^4) \tag{48}$$

and again the equations can't be consistent for an arbitrary renormalization coefficient C in $a_+(T_+) = Ca_-(T_-) \Rightarrow \tilde{a}_+(T_+) = C^{-1}\tilde{a}_-(T_-)$. There is however the remarkable exception corresponding to the permutation case $a_+(T_+) = \tilde{a}_-(T_-), \tilde{a}_+(T_+) = a_-(T_-) \Rightarrow C = \frac{a_+(T_+)}{\tilde{a}_+(T_+)} = \frac{\tilde{a}_-(T_-)}{a_-(T_-)}$. This is exactly the kind of discontinuity in time we have postulated within DG and we now see how a new kind of process, a discontinuous one, is made possible by our permutation symmetry while no such thing was even thinkable within GR!

Admittedly, in the GR case, the real reason behind the block was to force the continuity of densities and Hubble rates which was a quite arbitrary demand. In our theory, in [6] , H^2 and $\tilde{H}^2$ are always equal by definition while ρ and $\tilde{\rho}$ are equal at the crossing time T which makes the equation invariant under the exchange of the scale factors values at T as long as ρ, $\tilde{\rho}$, H^2 and $\tilde{H}^2$ remain unchanged. Inspection of Eq [6] alone therefore strongly suggests that the non arbitrary requirement is indeed the continuity of densities and squared Hubble rates rather than Hubble rates implying that the Hubble rate may either be continuous or flip sign at the transition and we shall keep open minded to this last possibility in the following sections.

Then however, the other Friedmann-DG equation [5] implies that the approximate equations of motion before and after the transition are $\dot{H} \approx -H^2/2 - 3Ka^2p$ and $\dot{\tilde{H}} \approx -\tilde{H}^2/2 - 3K\tilde{a}^2\tilde{p}$ respectively. Indeed, since the densities are continuous at the transition, so must be the pressures: $p_+ = p_-$, $\tilde{p}_+ = \tilde{p}_-$ and though pressures might not cross each other ($p_- \neq \tilde{p}_-$) at the same exact time the densities cross each other ($\rho_- = \tilde{\rho}_-$), we expect not so different pressures at this time insuring that the dominant source terms are still those multiplied by the greatest scale factor both before and after the transition which makes our approximations valid.

Then, since at any time $H = -\tilde{H} \Rightarrow \dot{H} = -\dot{\tilde{H}}$, $\tilde{H}_+^2 = \tilde{H}_-^2 = H_-^2$ we have:

$\dot{H}_+ = -\dot{\tilde{H}}_+ \approx \tilde{H}_+^2/2 + 3K\tilde{a}_+^2\tilde{p}_+ = H_-^2/2 + 3K\tilde{a}_+^2\tilde{p}_+ \approx -\dot{H}_- + 3K(\tilde{a}_+^2\tilde{p}_+ - a_-^2 p_-)$. But $\tilde{a}_+ = a_-$, $\tilde{p}_+ = \tilde{p}_-$ so eventually:

$$\dot{H}_+ \approx -\dot{H}_- + 3Ka_-^2(\tilde{p}_- - p_-) \tag{49}$$

This means that the time derivatives of the Hubble rates flip sign in very good approximation in a cold matter dominated universe and are therefore discontinuous at the transition. We cannot however exclude a very small contribution of pressures to this discontinuity, in case $p_- \neq \tilde{p}_-$. We see that there is no obvious physical motivation for requiring that the pressures should cross each other at T_- since a discontinuity of $\dot{H}$ is anyway unavoidable in contrast to the continuity of H^2. May be it's not really annoying to have a discrete symmetry only meaningful in the first Friedmann-DG equation because just as in GR, it is well known that this equation involving only first derivatives of the metric is rather a constraint that must be satisfied at any time than an evolution equation involving second derivatives of the metric as the second Friedmann-DG equation. Even in GR those equations don't have the same status (see [2] p163) and since our discontinuity only defines the new initial conditions for the subsequent evolution after the transition, it's natural that it is rather constrained by the first Friedmann-DG equation. However in the following we still want to require $p_- = \tilde{p}_-$ because then $\dot{H}_+ = -\dot{H}_- = \dot{\tilde{H}}_-$ is exact meaning that not only the H^2 but also the $\dot{H}$ are exchanged between the two sides at the transition so the complete geometrical terms of our equations as well (but not the H: we are still in an expanding universe)! Interestingly the two equations $p(T_-, V_-) = \tilde{p}(T_-, V_-)$, $\rho(T_-, V_-) = \tilde{\rho}(T_-, V_-)$ can have a solution if we have two or more free parameters : not only the time of the transition T_- but also extra-parameters defining the volume V_- of a spatial sub-domain of the universe in which the transition takes place.

From a phenomenological point of view the continuity of mean densities and pressures but also their perturbations insures that the discontinuous process itself has no observable effect at the time it occurs except two phenomena. First, following the transition the universe will start to accelerate: again the Hubble rate is continuous but not it's time derivative. Yet frequencies of clocks and light, energy levels of matter and radiation are cosmologically continuous from T_- to T_+: no unusual contribution to the redshifts. Second, the gravity from sources on our side (F fields perturbations) is expected to be almost switched off at the transition and as a consequence all our side stars should have exploded but we shall see later how this problem can be solved.

2.3.5. *A testable cosmological scenario*

The transition being triggered by equal densities and pressures on both sides of the Janus field, the dark side is also dust dominated at the transition and we also have the continuity of the Hubble rate[14]. This leads to a constantly accelerated universe $a(t') \propto t'^2$ in standard coordinate following the transition redshift.

Constraining the age of the universe to be the same as within LCDM the DG transition redshift can be estimated (see [15] equation 6) and confronted to the measured value $z_{tr} = 0.67 \pm 0.1$ within LCDM. The DG prediction is then $z_{tr} = 0.78$, very close to the LCDM value for a same universe age which is already encouraging.

The conjugate side being in contraction, should reach the radiative regime in the future, then our cosmological equation will simplify in a different way[h] :

$$\tilde{a}^2 \frac{\ddot{\tilde{a}}}{\tilde{a}} \approx \frac{4\pi G}{3} \tilde{a}^4 (\tilde{\rho} - 3\tilde{p}) = K \tilde{a}^2 \tag{50}$$

The solution $\tilde{a}(t) = C.sh(\sqrt{K}(t-t_0)) \approx C\sqrt{K}(t-t_0)$ for $1/C << \sqrt{K}(t-t_0) << 1$ so $a(t) \propto 1/(t-t_0)$ which translates into an exponentially accelerated expansion regime $e^{t'}$ in standard time coordinate.

We believe that our transition to a constantly accelerated universe is the most satisfactory alternative to the cosmological constant as it follows from first principles of the theory and eventually should fit the data without any arbitrary parameter, everything being only determined by the actual matter and luminous contents of the two conjugate universes, such content so far not being directly accessible for the dark side. More specifically, the parameter which replaces the cosmological constant in our framework is merely the redshift of densities equality i.e. the transition redshift z_{tr}. But in contrast to a cosmological constant which just corresponds to one possibility out of a myriad of other terms that one could add either on the left or the right of the Einstein equation without any strong theoretical motivation behind, hence implying a high degree of arbitrariness, everything in our framework

[h] That a quantity such as $\tilde{\rho} - 3\tilde{p}$ is expected to follow a $1/\tilde{a}^2$ evolution in the limit where all species are ultra-relativistic can be deduced from Eq (21)-(25) of [40] and the matter and radiation energy conservation equation rewritten as $\tilde{\rho} - 3\tilde{p} = 4\tilde{\rho} + \tilde{a}\frac{d\tilde{\rho}}{d\tilde{a}}$ in a radiation dominated dark side of the universe when $\tilde{\rho}$ and $\tilde{p} \approx 1/\tilde{a}^4(t)$.

follows from a different conceptual choice from the beginning: the existence of a non dynamical background which itself has strong theoretical motivations.

2.3.6. *Confrontation with the SN, Cepheids, BAO and CMB data*

In this section we now denote t the standard time rather than conformal time and present the detailed confrontation of our discrete transition scenario, the transition to a t^2 acceleration regime, with the most accurate current cosmological data: the cosmological microwave background spectrum, the Hubble diagram of Cepheid calibrated supernovae and baryonic acoustic oscillations.

Our purpose is therefore to determine the transition redshift between the $a(t) \propto t^{2/3}$ and $a(t) \propto t^2$ expansion laws (see Figure 5) allowing the best fit to those cosmological observable.

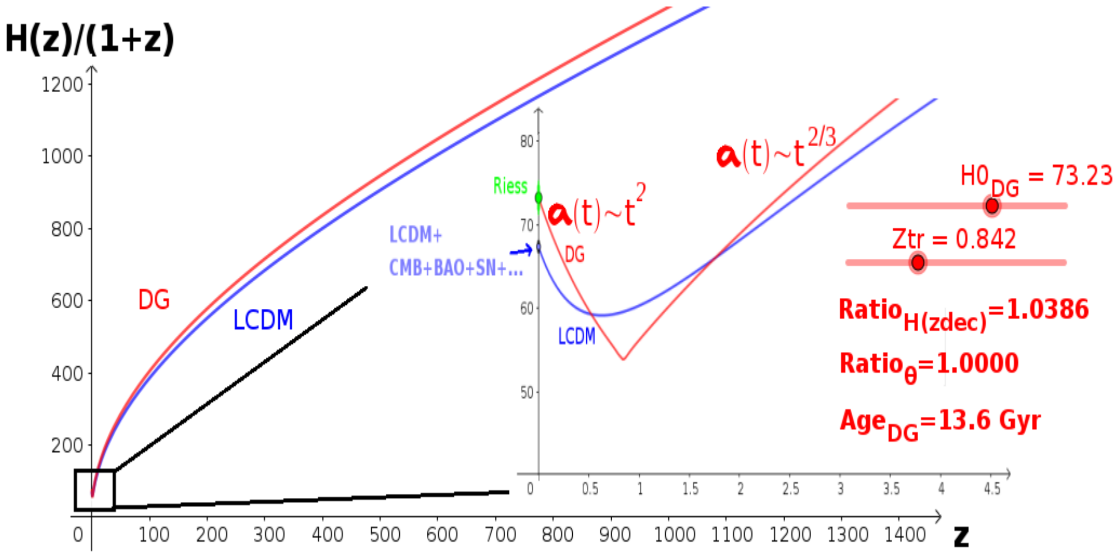

Fig. 5. A transition scenario vs the LCDM best fit

We already noticed a long time ago the remarkable (and not expected within LCDM) agreement between the supernovae Hubble diagram up to z=0.6 and a constantly accelerated universe [53] .ie. with $a(t) \propto t^2$ meaning a deceleration parameter q=-0.5. This is also confirmed by fig 2 from [54] with 740 confirmed SN IA of the JLA sample, some models fit functions (fig 2 bottom) even apparently indicating that our universe q(z) is asymptotically q=-0.5 at low redshift.

Just to confirm this tendency we use the same sample to fit α of a power law t^α evolution of the scale factor for redshifts restrained to the [0,z_{max}] interval and get:

$\alpha = 1.85 \pm 0.15$ for z_{max}=0.6 (one standard deviation from 2.)

$\alpha = 1.78 \pm 0.11$ for z_{max}=0.8; (two standard deviations from 2.)

As expected, beyond redshift 0.8 the power law is deviating from 2 by more than two sigmas : a hint that we may be reaching the decelerating $t^{2/3}$ regime in between redshift 0.6 and 0.8.

The next step is therefore to fit the transition redshift between a fixed $t^{2/3}$ and subsequent t^2 evolution laws, and we get: $z_{tr} = 0.67 + 0.24 - 0.12$ (Minos minimization method rather than Migrad should be used to get the reliable asymmetric errors, slightly smoothing the discontinuity in the derivative of H(z) with a sigmoid function also helps) with a $\chi^2 = 740.8$ slightly larger than that of the LCDM fit (739.4) but we notice by the way that allowing for two different normalization parameters on both sides of z_{tr} to account for possible imperfections of the inter-calibration of different instruments, thus an additional free parameter, the fit χ^2 is improved to 734.1 while z_{tr} is unchanged and the two normalization parameters are compatible (within 1 σ).

The un-binned residuals of the HD fit are shown in Figure 6 but showing either binned or un-binned information can be deceptive as the correlation between SN magnitudes is a crucial not shown information yet it can significantly influence the fit results, particularly when fitting DG. Also one should be warned that Malmquist bias corrections for selection effects have been applied to the JLA data assuming LCDM as the fiducial model. The parameters α and β fitted to minimize the distance modulus μ dispersion deviate from the LCDM values by less than the percent. The Pantheon data sample with 50% and 40% more SN at low and mid-high redshifts respectively was analyzed with the BBC method [75] improved for better bias corrections (but then it may be even more LCDM fiducial model dependent) giving similar results provided we take care of applying the re-calibration of the HST SNs as documented in section 2.4 of [76] for the JLA sample raw magnitudes (SALT2 output) implying a +57mmag offset relative to the raw magnitudes usually used in Pantheon for those same SNs. Our fit is particularly sensitive to this offset: not applying the re-calibration produces a more than one sigma deviation on the fitted transition redshift toward smaller values even though it concerns only a small number of high redshift SNs.

The next step is to use our Geogebra graphical tool to play with cursors and hopefully determine a z_{tr} value lying in the allowed interval according our previous SN fits, a H_0 close to the directly obtained value by Riess et all [55] (local distance ladder method through Cepheids and SN) and simultaneously allowing a good agreement to both the CMB data (angular position of first acoustic peak θ^* at decoupling and comoving sound horizon r_{drag}) [56] and BAO data (H(z), $D_M(z)$)[57].

We first of course need to correct the BAO data, obtained assuming the r_{drag} of a fiducial LCDM cosmology, to adapt them to our r_{drag}. Ω_{rad} is fixed as usual from the present day photon and neutrino densities. What's new is that Ω_M is then not anymore a free parameter. Indeed, we may define $\Omega_M(z_{tr}) = \frac{8\pi G\rho_M(z_{tr})}{3H_{tr}^2} = 1 - \Omega_r(z_{tr}) \approx 1$ since, beyond the transition redshift, we are indistinguishable from

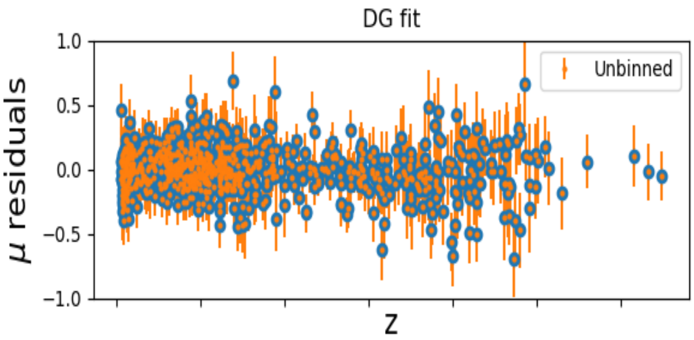

Fig. 6. Distance modulus un-binned residuals

a mere CDM flat cosmology without any dark energy nor cosmological constant. We can then extrapolate this to the usual present $\Omega_M = \frac{8\pi G\rho_M(0)}{3H_0^2}$ given that $\rho_M(z_{tr}) = \rho_M(0)(1+z_{tr})^3$ and $H_{tr} = H_0(1+z_{tr})^{1/2}$ for a constantly accelerated regime between z=0 and z=z_{tr}. Then, $\Omega_M = (1+z_{tr})^{-2}$.

Our attempts resulted in one of the best fits for $z_{tr} = 0.83$ (see Figure 7) for which we nevertheless cannot avoid a potential tension at the two sigma level for the lowest z D_M point (our prediction in the Angle(z) plot is the violet band: of course the ratio of distances is also the ratio of angles for a same given r_d but in our case r_d can be different and the data points would need the corresponding correction before we can compare them to our D_M prediction so we prefer to plot the ratio of Angles here) but we notice that this kind of tension appears almost unavoidable for any model that would fit the high H_0 value from Riess. The most likely origin of this tension is that linear regime perturbations from the contracting dark side start to grow differently than within LCDM after the transition redshift and as their gravity dominates over our side dark matter gravity as we shall see, those may deform the BAO peak in an unexpected way for those who analyze the data with LCDM as fiducial model to estimate various systematics. The results of transverse BAO measurements claimed to be less fiducial model dependent (using a 2d method) give a BAO angle systematically higher than the Planck-LCDM prediction at low redshifts implying a significantly better agreement with our predictions (see [77]Fig 6).

The small tension in H(z=0.7) corresponding to the full shape analysis of the BAO data remains acceptable but becomes more serious with the value obtained through reconstruction techniques [57] [61] [62], not only correcting various non-linear effects and reducing the errors but also assuming a growth rate of linear perturbations and correcting for redshift space distorsions (RSD) in a way which is valid for LCDM and not so wrong for wCDM but certainly not for Dark Gravity. Recent reanalysis using the linear point[86] in between the peak and the deep of the

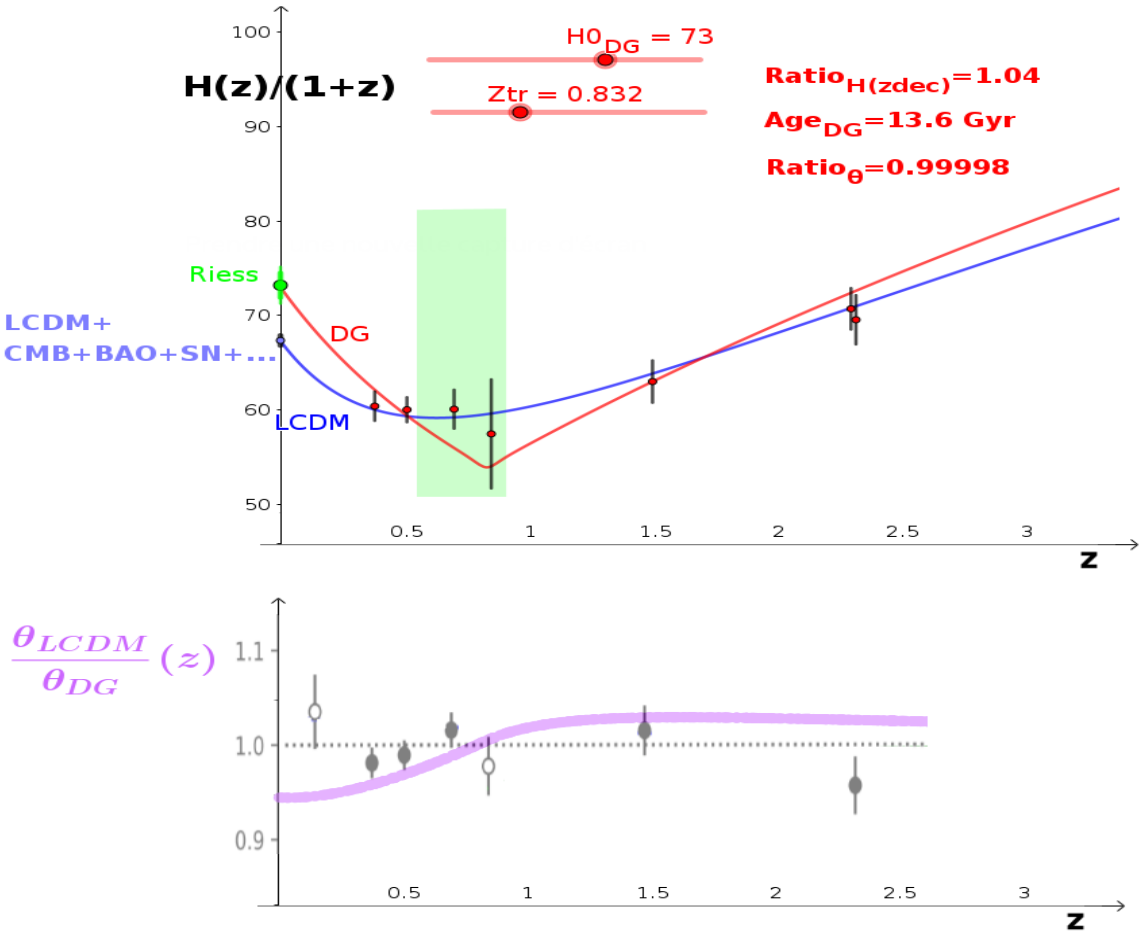

Fig. 7. A transition scenario confronted to CMB and BAO data, the violet band is our prediction for the $Angle(z)$ relative to the LCDM best fit prediction (horizontal line). The red data points in the H(z)/(1+z) plot are corrected for r_d DG cosmology and not expected to fit LCDM anymore. The green band is the allowed interval for the transition redshift (within 1 standard deviation) according our SN Hubble diagram fit

BAO claim that the usual BAO only method uncertainties are underestimated by a factor two while, as fiducial model independent as possible 2D methods, trying to measure the BAO scale in thin redshift slices to avoid systematics related to projection effects also often lead to significantly different BAO angles.

Recently [104] has strikingly confirmed that 2D BAO can tell us a very different story than 3D BAOs: "A comparison between the 3D BAO data, model dependent and obtained assuming LCDM, and the 2D BAO measurements, less model dependent, shows almost the same results for the LCDM scenario but completely different results for an Interactive Dark Energy model (allowing the decay of DM into DE) while giving strong evidence for the DE-DM coupling at more than 99 % CL, solving at the same time the H0 tension." There are several effects (wrong fiducial model choice related) that could explain the departure between 3D and 2D BAO: the conversion from redshift space to real space coordinates, reconstruction, the fitting template and RSD corrections.

The recent publication of DESy5 [108] supernovae results also gives much more support to DG H(z) scenario most probably because they are less affected by multi inter-calibration errors between surveys than were Pantheon: Combining DES-SN5YR and the CMB, they find (Ω_M , w0, wa) =(0.325+0.016 - 0.012 , -0.73 $\pm$ 0.11,-1.17-0.62 +0.55). This can result in a H(z) evolution very close to the one predicted by DG below the transition redshift (which effective w is -2/3 for DE and without DM in the accelerated era) while w0 is almost 2.5 sigmas away from the w=-1 of LCDM and the wa implies that w had to be much more negative ($w < -1$) at higher redshift, a thawing dark energy behaviour just as expected for a w0waCDM model trying to mimic DG H(z) with a very fast transition between DM and DE domination. Recent attempts to address the H0 but also the growth tensions, remarkably involve more elaborated composite or hybrid models than before with very different behavior before and after a transition redshift, so somehow approaching a solution closer to DG [114] [115] but their ability to solve the tensions are limited probably by significant systematics in the main cosmological probes (see my recent concatenated contributions to the Marcel Grossmann conference and CPPM weekly "showplot" meeting [111117] for a review on tensions and likely related underestimated systematics). Moreover there is growing evidence from many observables that we are not just facing a H0 tension between z=0 and the early universe but evolving H0(z) and S8(z) (see [117] and [116] and multi references therein from the same author) in the late universe, both ruling out a cosmological constant. Following the publication of the latest DESI BAO points, the situation has worsened as most main cosmological probes are in tension with each others so it does not really make sense any more to pursue the confrontation of our (or any other model) predictions and background measurements. Instead we shall focus in the future on the study of fluctuations through N-body simulations to try to understand which kind of systematical effects can be produced by the assumption of a LCDM fiducial cosmology for the fluctuations when DG is actually the correct theory.

Actually all current BAO analysis would need to be re-investigated within our framework. New BAO points at higher redshifts will prove crucial to eventually validate or rule-out our predictions, given that on the other hand, before the transition redshift, understanding the growth of fluctuations in our framework is much easier than after the transition redshift.

The confrontation with Big Bang nucleosynthesis data is also granted to be successful given how close to the LCDM one is our H(z) at high redshift (Figure 5). Our r_d is only less than three percent lower than the LCDM one and the age of the Universe is still reasonable given the oldest stars ages (13.6 billion years) : this is because the much higher than LCDM H(z) that we have at low redshifts is compensated by a much lower H(z) than LCDM between 0.6 and 1.6 (Figure 5) : needless to say that this good property is not shared by most models trying to solve the H_0 tension with new physics at low redshifts.

However our value H(zdec) at the redshift of decoupling is 4% over the LCDM-Planck value and it is currently admitted that a good fit to the Planck power spectra requires H(zdec) to not deviate by significantly more than 1% from the LCDM best fit value.

A solution would be a mechanism increasing the density of matter as the decelerating universe approaches the transition redshift: in the reverse way, for increasing redshifts, the total density and therefore H(z) would decrease relative to what is expected from the usual a^{-3} law for the matter density and would hopefully bring H(zdec) closer to the Planck constraint. The transition of massive neutrinos from relativistic to non relativistic regime as they cool down produces exactly this kind of effect and indeed a sum of neutrino masses near the 800 meV upper bound provided by direct detection experiments can bring our H(zdec) to within 2% of the LCDM value. It would thus appear that the same Planck power spectra that usually constrain the sum of neutrino masses to be small within LCDM, at the contrary favour the highest possible neutrino masses within DG.

However such neutrino masses are not sufficient to completely solve the tension with Planck and introduce yet another one in the matter power spectrum: the well known power suppression on small scales for massive neutrinos with respect to massless neutrinos. Indeed, the current upper bound from Lyman-α measurements alone at $2 < z < 4$ is $\Sigma m_\nu < 0.71 eV$ (95% CL) [80].

There remains the amazing possibility that the neutrino mass generation is linked to physics at the transition redshift. As [79] reminds us, "it seems striking that the energy scales of dark energy and neutrino masses are numerically very close, $\rho_\Lambda^{1/4} \approx m_\nu \approx meV$. If cosmological data permit larger neutrino masses during dark energy domination, there could be an intriguing theoretical connection between these two phenomena", and this has motivated many theoretical proposals (see references therein). In [79] it is shown how the usual cosmological constraints from BAO, SN and Planck power spectra on neutrino masses are relaxed if neutrinos acquire their mass at low redshifts, this being a supercool transition as it would occur at much lower redshifts than the usual relativistic to non relativistic transition redshift for large neutrino masses. Such constraints are even more relaxed in our case provided the effect of this mass generation would take place between z=2 and z=10 just in such a way as to not disturb the already successful confrontation of DG with SN and BAO data at lower redshifts. The total sum of neutrino mass can even exceed limits from direct detection experiments if the masses of the sterile neutrinos (the right handed ones) are generated in the same process before the transition redshift. Indeed current constraints on the effective number of relativistic degrees of freedom N_{eff} do not forbid a small contribution of sterile neutrinos to N_{eff} and those neutrinos may produce a large density, hence H(z), increase at the time they acquire mass since these masses are unconstrained and could be much larger than active neutrinos masses. Of course neutrinos cannot acquire mass for free and it is often the energy of a postulated scalar field which is transmitted to the neutrinos in such transition proposals. It remains to be investigated whether an other kind of energy source, specific to DG, could as well make the job. At last we can mention for completeness that many models have been proposed to alleviate the H_0 tension by increasing H(z) along with a reduction of r_d at early times (for instance a recent proposal is to increase N_{eff} and to compensate this by a lepton asymmetry in the neutrino sector, i.e. 2% more neutrinos than anti-neutrinos to avoid conflicts with BBN observations [82] without modifying the behaviour of dark energy at late times and this allows a 2% increase of H_0). Such ideas as well could help us as we need an H(z) increase before our transition redshift.

An even more likely scenario within DG is the following: when the global densities are close to crossing each others, we expect something like what is represented in Fig. 8. We can see that we have two domains: the brown-yellow area (1) in which the mean density on our side dominates the dark side mean density over the whole domain that progressively shrinks in time and the blue one (2) in which the mean density dominance is reversed, that progressively widens. It might be that the cosmological equations actually apply in these restricted domains before and after the transition. For instance when domains (1) shrinks it loses it's lower density regions so it's mean global density decreases slower than in the case of a global domain including the whole universe, and this is all we need to improve the fit to Planck data (density and H(z) increase as we approach the transition redshift). Notice that now the still discontinuous transition (relay) is not only between our and the dark sector but also between domain (1) and (2) i.e. when $\rho_1 = \tilde{\rho_2}$. Of course we also simultaneously require $\rho_2 = \tilde{\rho_1}$ for the transition to be triggered. This becomes possible as domains (1) and (2) are not uniquely defined as only the mean density and not the local density is at play. Before the transition, (1) is the cosmological domain and the background in (2) is static in Fig. 8, while the situation is reversed after the transition.

A simpler alternative is still the case of a single domain with the transition triggered by $\rho = \tilde{\rho}$ and the domain limits at transition determined by the condition $p = \tilde{p}$. If this condition progressively excludes lowest density regions from the domain, again H(z) can increase as we approach the transition redshift. However we don't understand in this case why high density regions including stars also don't belong to the domain (otherwise their gravity would be switched off) so we need the additional constraint that the domain should also not include the very high density regions such as stars. Eventually excluding both low density and highest density regions should still increase the mean density of the domain to increase H(z) as we approach the transition redshift so this alternative though simpler may be is less natural.

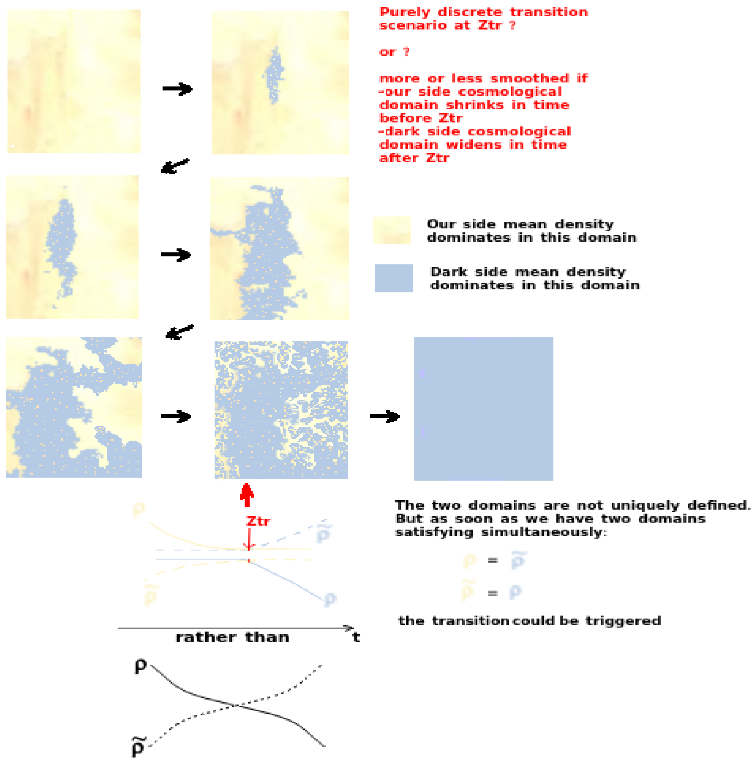

Fig. 8. Density maps before and after Z_{tr}

Out of many possibilities that work equally well, a completely ad hoc example is illustrated in Figures 9,10 and 11. Here we have assumed the usual lower bound (60 meV) for the active neutrinos. To produce a nearly 30% increase in the total matter density either the mechanism described in Fig. 8 is at work or sterile neutrinos should acquire their mass near z=2 just as needed to allow this 30% increase. Unsurprisingly, the resulting TT+TE+EE power spectra fit (obtained thanks to a Class code [78] suitably modified for our needs) $\Delta\chi^2$ is only +4.4 relative to LCDM. Only the very large scale TT and $\phi\phi$ power spectra are significantly sensitive to the effect of density fluctuations at redshifts lower than z_{tr} and since at such redshifts much work remains to be done to properly simulate the highly non trivial interactions between our and the dark side fluctuations at various scales, a simplified first step methodology was adopted consisting in the assumption of a homogeneous dark energy fluid that would produce the same H(z) as DG in the accelerated universe. For this reason the TT and $\phi\phi$ power spectra obtained on the largest scales should only be considered as indicative of what we can expect from such naive assumption.

We already mentioned a serious problem with our transition to acceleration: if

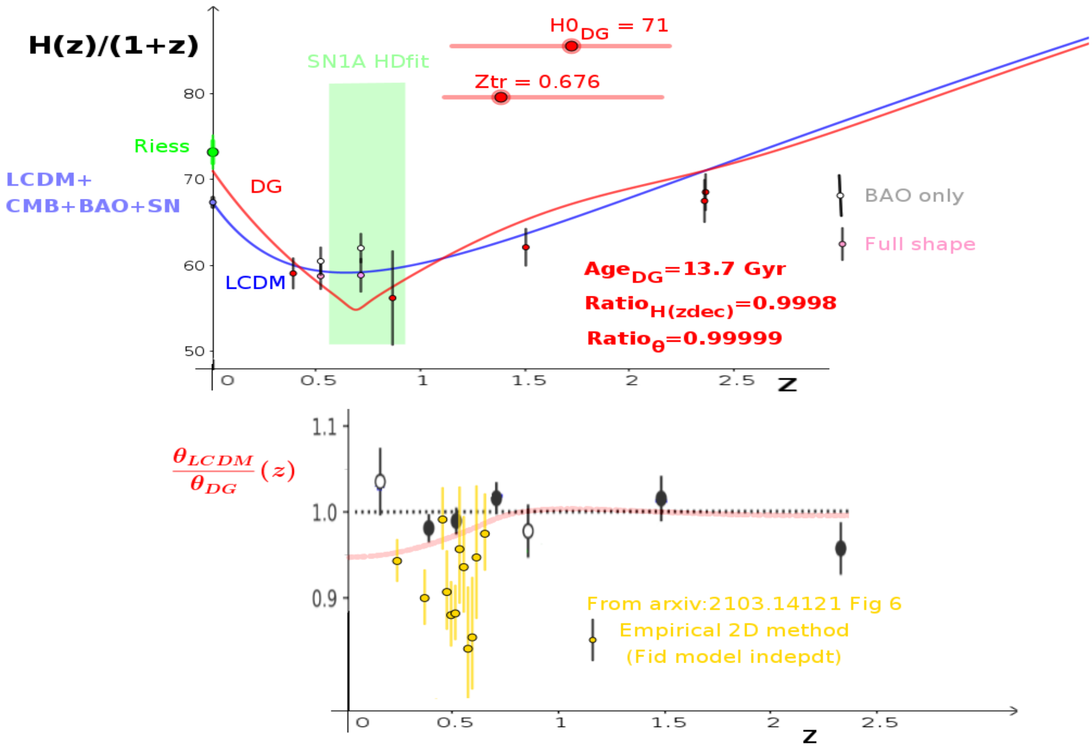

Fig. 9. Corrected DG confronted with BAO scale measurements, SN and Planck constraint on H(zdec), In orange are reported measurements using a less fiducial model dependent 2D method

it applies to the whole universe then gravitationally bound systems such as stars should have disappeared on our side following this transition. The inescapable consequence is that the transition did not actually take place over the whole universe but only over a sub-domain excluding very non linear structures areas, therefore implying that we not only have the postulated discontinuity in time at the transition but also discontinuities in space at the frontier of small domains in which the transition did not occur so that as explained above at least stars and planet should remain gravific to insure their own stability in such domains (the dark side will not help for that as we shall soon check).

Therefore the cosmological domain (the ocean) over which we apply the cosmological equations that determine the background evolution before and after crossing excludes the island domains : those in which the transition from C to 1/C did not occur allowing them to keep their gravitational strength as required and this is represented in Figure 12. The good new is that this feature is also a prediction of our improved transition scenario to better fit Planck data. Indeed, near z_{tr}, Figure 8 shows the areas that are still not in domain 2: the cosmological domain that now drives the evolution of the scale factors. These islands are presumably where non

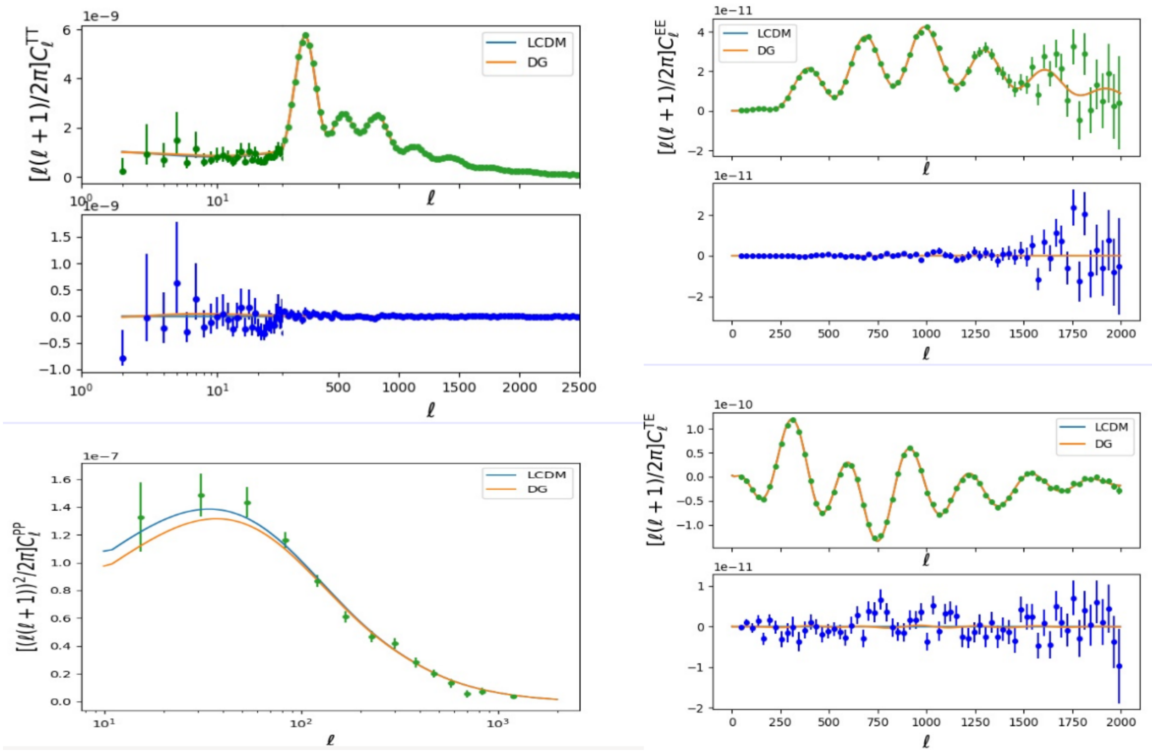

Fig. 10. corrected DG confronted with Planck and LCDM predicted Power spectra

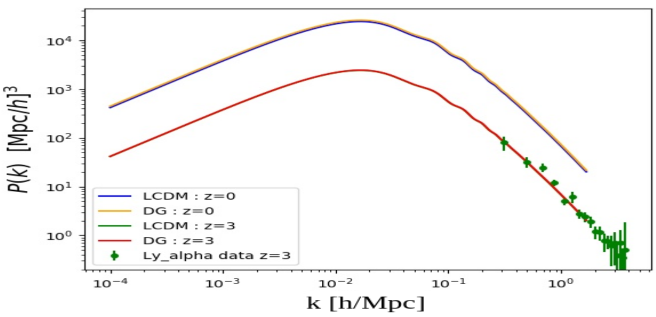

Fig. 11. corrected DG confronted with Lyman-alpha (at mean z =3) Matter Power spectra and LCDM predicted Power spectra at z=0 and z=3

linear structures of matter did not yet lose their gravity, until now, but certainly will in the future before the end of the cosmological cycle.

But to be in position to better understand the ocean and islands we now need

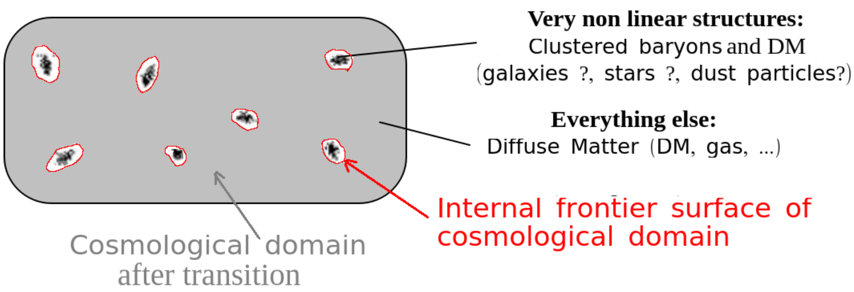

Fig. 12. Cosmological domain with internal frontiers

to investigate the local solutions (Schwarzschild and gravitational waves) of our field equations, then the rules that apply at domain frontiers. Later we shall also establish equations and solutions for the evolution of linear fluctuations.

Investigating the possibility of a similar transition in the no cosmological constant version of DG ($\zeta \propto 1/a^4$), we realize that the permutation of the scale factors imply the corresponding one for the ζ variables, that nearly just before the crossing of global densities triggering the permutation the dark side is already significantly gravific which could boost the formation of structures at high redshift, but that a serious drawback is that the crossing of global densities should produce an exact cancellation resulting in vanishing Hubble rate at transition. This is of course excluded by the data but the scenario with two domains explained above might save this version of the theory: in a given domain at transition the densities can still be very different as we require $\rho_1 = \tilde{\rho}_2$ and $\rho_2 = \tilde{\rho}_1$ rather than $\rho = \tilde{\rho}$.

3. Isotropic solution about a common Minkowski background

We are now interested in the isotropic solution in vacuum (equivalent of the GR Schwarzschild solution) of the form $g_{\mu\nu} = (-B, A, A, A)$ in e.g. $d\tau^2 = -Bdt^2 + A(dx^2 + dy^2 + dz^2)$ and $\tilde{g}_{\mu\nu} = (-1/B, 1/A, 1/A, 1/A)$ about a common Minkowski background. This means that such a solution is expected to be valid at t=0 when the two scale factors were the same. We get:

$$A = e^{\frac{2MG}{r}} \approx 1 + 2\frac{MG}{r} + 2\frac{M^2G^2}{r^2} \tag{51}$$

$$-B = -\frac{1}{A} = -e^{\frac{-2MG}{r}} \approx -1 + 2\frac{MG}{r} - 2\frac{M^2G^2}{r^2} + \frac{4}{3}\frac{M^3G^3}{r^3} \tag{52}$$

perfectly suited to represent the field generated outside an isotropic source mass M. This is different from the GR one, though in good agreement up to Post-Newtonian order. The detailed comparison will be carried out in section 6. It is straightforward to check that this Schwarzschild new solution involves no horizon. The solution also confirms that a positive mass M in the conjugate metric is seen as a negative mass -M from its gravitational effect felt on our side.

4. Equations and stability of the background

4.1. *About a common Minkowski background*

The equations about a common Minkowskian background i.e. with $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ and $\tilde{g}_{\mu\nu} = \eta_{\mu\nu} + \tilde{h}_{\mu\nu}$, $\tilde{h}_{\mu\nu} = -h_{\mu\nu} + h_{\mu\rho}h_{\nu\sigma}\eta^{\rho\sigma} + O(3)$, look the same as in GR, the main differences being the additional dark side source term $\tilde{T}_{\mu\nu}$ and an additional factor 2 on the lhs:

$$2(R^{(1)}_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}R^{(1)\lambda}_{\lambda}) = -8\pi G(T_{\mu\nu} - \tilde{T}_{\mu\nu} + O(hT) + t_{\mu\nu} - \tilde{t}_{\mu\nu}) \tag{53}$$

where we have isolated as usual the linear part in which the linear part of the Ricci tensor in $h_{\mu\nu}$ is as usual indicated by the (1) superscript (also notice the O(hT) terms arising because we can't anymore simplify $\sqrt{g} = 1 + O(h)$ factors multiplying the matter energy-momentum tensors as in GR.) and therefore the linearized ($|h_{\mu\nu}| << 1$) equations are similar to GR ones:

$$2(R^{(1)}_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}R^{(1)\lambda}_{\lambda}) = -8\pi G(T^{(0)}_{\mu\nu} - \tilde{T}^{(0)}_{\mu\nu}) \tag{54}$$

and exactly the same in vacuum ($\Box h_{\mu\nu} = 0$).

However to second order in $h_{\mu\nu}$ (plane wave expanded as usual) we found that the only non canceling contributions to $t_{\mu\nu} - \tilde{t}_{\mu\nu}$ on the rhs, vanish upon averaging over a region of space and time much larger than the wavelength and period (this is the way the energy and momentum of any wave are usually evaluated according [2] page 259). This $t_{\mu\nu} - \tilde{t}_{\mu\nu}$ is standing as usual for the energy-momentum of the gravitational field itself because the Linearized Bianchi identities are still obeyed on the left hand side and it therefore follows the local conservation law:

$$\frac{\partial}{\partial x^{\mu}}(T^{\mu\nu} - \tilde{T}^{\mu\nu} + t^{\mu\nu} - \tilde{t}^{\mu\nu} + O(hT)) = 0 \tag{55}$$

We can try to go beyond the second order noticing that the DG equation (3) has the form $X^{\mu\nu} - \tilde{X}^{\nu\mu} = -8\pi G(Y^{\mu\nu} - \tilde{Y}^{\nu\mu})$ and can be split in a $\mu \leftrightarrow \nu$ symmetric, $X^{\mu\nu}_s - \tilde{X}^{\mu\nu}_s = -8\pi G(Y^{\mu\nu}_s - \tilde{Y}^{\mu\nu}_s)$, and a $\mu \leftrightarrow \nu$ anti-symmetric $X^{\mu\nu}_a + \tilde{X}^{\mu\nu}_a = -8\pi G(Y^{\mu\nu}_a + \tilde{Y}^{\mu\nu}_a)$, in which the s (resp a) indices refer to the symmetric (resp anti-symmetric) parts of the tensors. Though the antisymmetric equation could in principle source gravitational waves, its production rate is expected to be extremely reduced vs GR because the dominant source term is at most of order hT rather than T in the Y term.

The value of the $\mu \leftrightarrow \nu$ symmetric equation is the manifest anti-symmetry of its lhs under the permutation of $g_{\mu\nu}$ and $\tilde{g}_{\mu\nu}$. Replacing $g_{\mu\nu} = e^{\bar{h}_{\mu\nu}}$ thus $\tilde{g}_{\mu\nu} = e^{-\bar{h}_{\mu\nu}}$, this translates into the odd property of the lhs to all orders in $\bar{h}_{\mu\nu}$. Then we are free to use the plane wave expansion of this new $\bar{h}_{\mu\nu}$ (not to be confused with $h_{\mu\nu}$ nor $\tilde{h}_{\mu\nu}$) instead of $h_{\mu\nu}$ and because each term of the perturbative series has an odd number of such $\bar{h}$ factors, such term will always exhibit a remaining $e^{i\mathbf{kx}}$ factor which average over regions much larger than wavelength and period vanishes (in contrast to [1] page 259 where the computation is carried on for quadratic terms for which we are left with some x^{μ} independent, hence non vanishing, cross-terms).

Our new interpretation is that any radiated wave of this kind (sourced from the symmetric rather than the anti-symmetric part of the equation) will both carry away a positive energy in $t^{\mu\nu}$ as well as the same amount of energy with negative sign in $-\tilde{t}^{\mu\nu}$ about Minkowski resulting in a total vanishing radiated energy. Thus the DG theory, so far appears to be dramatically conflicting with both the indirect and direct observations of gravitational waves.

4.2. *Classical stability about an evolved background*

But actually, the asymptotic behaviors of the two sides of the Janus field are only the same at t=0, so we now need (next section) to investigate the case of different asymptotic values corresponding to what we expect after the scale factors have evolved.

After the background has evolved even slightly, we can write $g_{\mu\nu} = C^2 e^{\bar{h}_{\mu\nu}}$ and $\tilde{g}_{\mu\nu} = C^{-2} e^{-\bar{h}_{\mu\nu}}$ to study the evolution of a small fluctuation relative to the background but varying faster than the background (this is why the scale factor can be approximated by the constant C) otherwise it would not be a menace to the stability of this background. Then it's not necessary to go beyond the second order in $\bar{h}_{\mu\nu}$ to derive: $t^{\mu\nu} - \tilde{t}^{\mu\nu} \approx C^6 t^{\mu\nu}_{C=1} - C^{-6}\tilde{t}^{\mu\nu}_{C=1} = (C^6 - C^{-6})t^{\mu\nu}_{C=1}$. Since the expression of $t^{\mu\nu}_{C=1}$ is the same as in GR, the energy it carries is positive and so is the energy carried by $t^{\mu\nu} - \tilde{t}^{\mu\nu}$ when $C > 1$ (resp negative for $C < 1$) i.e. when the dominant source term also has a positive (resp negative) total energy density (remember that for small fluctuations the matter fluctuation must also be small relative to the background density). As for the weaker source term (it has both a smaller density and .eg. it's coupling is weakened by a factor $1/g = 1/C^8$ for $C > 1$), it is for this reason not the one that mainly drives the evolution of the fluctuation so it is harmless at a classical level even though it has the wrong sign.

This shows the classical stability of the background under small fluctuations at any time. Only in the strong gravity domain (when we approach the Schwarzschild radius) should we reinvestigate the stability of small classical fluctuations about the new background (the DG Asymptotically $C\eta$ Schwarzschild solutions) because as we shall see later at some point the geometrical terms from both sides in the field equation become again of the same order when we get close enough to the Schwarzschild radius (see the behaviour of the numerically solved solution in Figure 13). However even in such case at least the classical instabilities may be harmless as they take place beyond the pseudo-horizon of our pseudo Black Hole. Indeed gravity might be switched off (and the pseudo black hole destroyed) on our side on a time scale smaller than the extremely dilated timescale of such instabilities from the outside observer point of view. We also notice that any way even if the ghost classical interaction were to become dominant, such ghosts are in general not considered catastrophic at a classical level but only at the quantum level ([37] : "for ghosts, background is QM unstable but classically stable") so our ultimate protection will be that DG should remain a semi-classical theory of gravity.

5. Differing asymptotic values

5.1. *The C effect*

Due to expansion on our side and contraction on the dark side the common Minkowskian asymptotic value of our previous section is actually not a natural assumption in the present universe. At the contrary a field assumed to be asymptotically $C^2\eta_{\mu\nu}$ with C constant (here we neglect the evolution of the background as usual in the very non linear regime) has its conjugate asymptotically $\eta_{\mu\nu}/C^2$ so their asymptotic values should differ by many orders of magnitude. Given that $g^{C^2\eta}_{\mu\nu} = C^2 g^{\eta}_{\mu\nu}$ and $\tilde{g}^{\eta/C^2}_{\mu\nu} = \frac{1}{C^2}\tilde{g}^{\eta}_{\mu\nu}$, where the $< g^\eta, \tilde{g}^\eta >$ Janus field is asymptotically η, it is straightforward to rewrite the local DG Janus Field equation now satisfied by this asymptotically Minkowskian Janus field after those replacements. Hereafter, we omit all labels specifying the asymptotic behavior for better readability and only write the time-time equation satisfied by the asymptotically $\eta_{\mu\nu}$ Janus field.

$$C^2\sqrt{g}\frac{G_{tt}}{g_{tt}} - \frac{1}{C^2}\sqrt{\tilde{g}}\frac{\tilde{G_{tt}}}{\tilde{g}_{tt}} = -8\pi G(C^4\sqrt{g}\delta\rho - \frac{1}{C^4}\sqrt{\tilde{g}}\tilde{\delta\rho}) \tag{56}$$

where $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$ and $\delta\rho$ is the energy density fluctuation for matter and radiation. The tilde terms again refer to the same tensors except that they are built from the corresponding tilde (dark side) fields.

Then for $C >> 1$ we are back to $G_{tt} = -8\pi G C^2 g_{tt}\delta\rho$, a GR like equation for local gravity from sources on our side because all terms depending on the conjugate field become negligible on the left hand side of the equation while the local gravity from sources on the dark side is attenuated by the huge $1/C^8$ factor (in the weak field approximation, $G_{tt} = 8\pi G\frac{\delta\tilde{\rho}}{C^6}$). From $g^{\eta}_{\mu\nu}$ we then can get back $g^{C^2\eta}_{\mu\nu}$ and of course absorb the C constant by the adoption of a new coordinate system and redefinition of G, so for $C >> 1$ we tend to GR : we expect almost the same gravitational waves emission rate and almost the same weak gravitational field. However on the dark side everything will feel the effect of the anti-gravitational field from bodies on our side amplified by the same huge factor relative to the gravity produced by bodies on their own side.

The roles are exchanged in case $C << 1$. Then the GR equation $\tilde{G}_{tt} = -\frac{8\pi}{C^2}G\tilde{g_{tt}}\delta\tilde{\rho}$ is valid on the dark side while the anti-gravity we should feel from the dark side is enhanced by the huge $1/C^8$ factor relative to our own gravity (given in the weak field approximation by solving $\tilde{G}_{tt} = 8\pi G C^6\delta\rho$ for $\tilde{g}_{\mu\nu}$ from which we derive immediately our side $g_{\mu\nu}$ of the Janus field).

Only in case C=1 do we recover our local exponential Dark Gravity, with no significant GW radiations and also a strength of gravity ($G_{tt} = -4\pi G\delta\rho$) reduced by a factor $2C^2$ relative to the above GR gravity ($G_{tt} = -8\pi G C^2\delta\rho$).

It's important to stress that the phenomenology following from different asymptotic behaviours of the two faces of the Janus field here has no peer within GR in which a mere coordinate transformation is always enough to put the gravitational field in an asymptotically Minkowskian form in which a redefinition of the gravitational constant G gives back the usual gravitational potentials. This would still be possible in DG for one face of the Janus field but not for both at the same time. The new physics emerges from their relative asymptotic behaviour which can't be absorbed by any choice of coordinate system[i].

5.2. *Frontier effects*

We are here interested in specifying the kind of effects related to the occurrence of C and 1/C asymptotic gravity spatial domains and more specifically at the frontier between two such domains. We already explained earlier why we think that such configuration actually occurs.

Let's assume a 1/C asymptotic domain neighbouring a C asymptotic domain and a weak field so that we can for instance approximate the g_{00} metric element by an exponential function. Let's assume we have point masses M_1 on our side and M_2 on the dark side, both being in the C domain (of our side metric). Then according the previous section results, we have :

$$g_{00} \approx C^2 e^{-G(C^2 M_1/r_1 - C^{-6} M_2/r_2)} \tag{57}$$

anywhere in the C domain at distance r_1 from M_1 and r_2 from M_2.

Switching from a formula like (56) valid for density fluctuations to a formula valid for point mass sources as we just did requires justification. We may notice that, when C is not anymore a constant but a genuine scale factor, in order to recover the Mac Vittie metric behaviour for g_{00}, which is considered to be the best effort metric in GR when the source is a point mass in a perfect fluid with homogeneous density, a useful trick is to replace $\delta\rho$ by M/C^3 instead of just replacing $\delta\rho$ by M as we did. Then of course one should replace $\delta\tilde{\rho}$ by $\tilde{M}/\tilde{C}^3$ and the g_{00} metric element formula would rather be:

$$g_{00} \approx C^2 e^{-G(C^{-1} M_1/r_1 - C^{-3} M_2/r_2)} \tag{58}$$

Since the dominance relationships among the two terms are not modified in this new formula with respect to the former, our qualitative results will not be modified in the sense that the negligible terms will be the same so in the following we stick to the first formula. Indeed a bit of caution is not superfluous as the $ra(t)$ dependency of the Mac Vittie metric is related to the questionable requirement that there should be no radial flow, no energy accretion toward the mass in such solution: this is in line with the perfect fluid hypothesis hence a vanishing Einstein tensor element G_{tr} and this in turn requires a non homogeneous pressure to resist the accretion. In fact this question brings us back to an open and difficult problem in GR : how to correctly

[i]For $C >> 1$ we also even better recover the gauge invariance of GR, meaning that the scalar and vector degrees of freedom tend to decouple even more, leaving the pure tensor modes as in GR

describe the metric of an isotropic mass in an homogeneous expanding background which we do not claim to solve here. Moreover , we actually never have an isotropic mass in an homogeneous fluid in realistic situations such as for a star: in the solar system for instance even the baryonic density alone in the sun neighbourhood is orders of magnitude greater than the critical density and decreases as $1/r^2$.

Anyway what matters for us is that the g_{00} metric element can be extended anywhere in a neighboring 1/C domain by

$$g_{00} \approx C^{-2} e^{-G(C^2 M_1/r_1 - C^{-6} M_2/r_2)} \tag{59}$$

In other words the metric is simply renormalized by a constant factor at the frontier between two domains. Now let's assume we have two point masses, M_3 on our side and M_4 on the dark side, both being in the 1/C domain (of our side metric). Then we get:

$$g_{00} \approx C^{-2} e^{-G(C^{-6} M_3/r_3 - C^2 M_4/r_4)} \tag{60}$$

anywhere in this 1/C domain at distance r_3 from M_3 and r_4 from M_4. Again this can be extended anywhere in the neighbouring C domain by

$$g_{00} \approx C^2 e^{-G(C^{-6} M_3/r_3 - C^2 M_4/r_4)} \tag{61}$$

At last if we both have the previous two couples of masses we can merely combine the above results in the C domain to get:

$$g_{00} \approx C^2 e^{-G(C^2(M_1/r_1 - M_4/r_4) + C^{-6}(M_3/r_3 - M_2/r_2))} \approx C^2 e^{-G(C^2(M_1/r_1 - M_4/r_4))} \tag{62}$$

and in the 1/C domain to get:

$$g_{00} \approx C^{-2} e^{-G(C^2(M_1/r_1 - M_4/r_4) + C^{-6}(M_3/r_3 - M_2/r_2))} \approx C^{-2} e^{-G(C^2(M_1/r_1 - M_4/r_4))} \tag{63}$$

the last approximations being for $C >> 1$. We realize that in both domains the strengths of gravity and anti-gravity respectively from M_1 and M_4 are the same! The above combination reflects our intuition that the frontier surface behaves as a secondary source (Huygens principle) when it propagates (renormalizing it in passing) the field from one domain to the neighboring one so that eventually in a given domain the fields from masses in any domains, non linearly mix just as in GR.

Now that we have clarified how the metric transforms at domain frontiers it just remains to clarify how the matter and radiation fields behave there. Just as the discontinuity in time of the scale factor triggering the acceleration of the universe had no effect on densities, the discontinuity in space from C^2 to C^{-2} implied by the different normalization between the two domains (itself implied by the scale factors permutation) is again required not to affect the energy levels of particles crossing the frontier and their associated densities.

6. Back to Black-Holes and gravitational waves

6.1. *Back to Black-Holes*

Let's consider the collapse of a massive star which according to GR should lead to the formation of a Black Hole. As the radius of the star approaches the Schwarzschild radius the metric becomes singular there so the process lasts an infinite time according to the exterior observer. If the local fields both outside and inside the star have huge asymptotic C values, we already demonstrated that the gravitational equations tend to GR. However this can't be the case when we approach the Schwarzschild radius because C is finite and the metric elements can grow in such a way that we could not anymore neglect the dark side geometrical term. Therefore presumably the horizon singularity is avoided as well for C $\neq 1$. To check this we need the exact differential equations satisfied in vacuum by C-asymptotic isotropic static metrics of the form $g_{\mu\nu} = (-B, A, A, A)$ in e.g. $d\tau^2 = -Bdt^2 + A(dx^2 + dy^2 + dz^2)$ and $\tilde{g}_{\mu\nu} = (-1/B, 1/A, 1/A, 1/A)$. With $A = C^2 e^a$ and $B = C^2 e^b$, we get the differential equations satisfied by a(r) and b(r):

$$a'' + \frac{2a'}{r} + \frac{a'^2}{f} = 0 \tag{64}$$

$$b' = -a' \frac{1 + a'r/f}{1 + 2a'r/f} \tag{65}$$

where $f = 4\frac{e^{a+b}C^4+1}{e^{a+b}C^4-1}$. GR is recovered for C infinite thus f=4. Then the integration is straightforward leading as expected to

$$A = (1 + U)^{f=4}; \tag{66}$$

$$B = (\frac{1-U}{1+U})^{(f=4)/2} \tag{67}$$

where $U = GM/2r$ and the infinite C can be absorbed by opting to a suitable coordinate system : then there is no dark side. DG C=1 corresponds to b=-a, f infinite and the integration, as expected, gives $A = e^U$, $B = e^{-U}$.

The integration is far less trivial for intermediary Cs because then f is not anymore a constant. Moreover, in addition to the two above equations derived from the DG tt, and rr equations there is now a third additional differential equation derived from the $\theta\theta$ or $\phi\phi$ DG equations :

$$a'' + b'' + \frac{a'}{r} + \frac{b'}{r} + \frac{2b'^2}{f} = 0 \tag{68}$$

we can simplify this equation in terms of a alone using the two other equations and for a finite f (for f infinite i.e. C=1 all the equations are consistent), we find the constraint

$$\frac{a'^2(\frac{1}{f^2} - \frac{1}{16})}{1 + 2\frac{a'r}{f}} = 0 \tag{69}$$

which is only automatically satisfied for f=4 (GR). So again we have no solution except the trivial Minkowski one because it turns out that isotropy alone already two much restricts our number of degrees of freedom with respect to our number of independent DG equations. Again our solution is to introduce an offshell, here r dependent scalar just by letting for instance the coupling constants G_R and $\tilde{G}_{\tilde{R}}$ multiplying our two Einstein Hilbert actions vary assuming them to be related by $G_R = \frac{1}{\tilde{G}_{\tilde{R}}}$. Then $f = 4\frac{e^{a+b}G_R C^4+1}{e^{a+b}G_R C^4-1}$ and the constraint is replaced by a third differential equation for G_R:

$$G'_R = \frac{G_R}{2}\frac{ra'^2}{f+2a'r}(\frac{1}{f^2}-\frac{1}{16}) \tag{70}$$

In the numerical integration we shall find that G_R is almost spatially constant even when C is not that big (10^3) and therefore has an extremely small influence on the solution today even though this or other equivalent mechanism seems mandatory. The offshell G_R that we let vary now is not the same that we varied to get our cosmology. The latter was rather introduced inside the matter actions while the new one is rather in the Einstein Hilbert actions. Actually the G_R that we let vary now would also be efficient to unfreeze the cosmology. But instead of introducing this new "variable constant" G_R in place of the initial one G we can alternatively make another diagnostic: when we solve for a Schwarzschild like solution, it's the fact that we are in vacuum outside the source that prevents any possibility to instead let vary in space the G that we already used to unfreeze a solution for our cosmological equations and now use it to also insure a realistic isotropic solution as well. But in realistic conditions there is no perfect vacuum (for instance the CMB radiation is everywhere) so presumably the varying constant G(r) could play it's role fine while when the density tends to zero the needed coupling is expected to diverge in which case we might have an instability triggering either a jump to a C=1 solution or to a C infinite solution (i.e. complete decoupling of the Dark Side) in such empty outside the source domain. To check that a varying G alone is enough so that G_R is superfluous let's write the DG equations still in the static isotropic case but now inside matter with density $\rho(r)$ and pressure p(r).

$$a'' + \frac{2a'}{r} + \frac{{a'}^2}{f} = -8\pi G_0 \hat{\rho} \tag{71}$$

$$\frac{a'}{r} + \frac{b'}{r} + (a' + 2b')\frac{a'}{f} = 8\pi G_0 \hat{p} \tag{72}$$

$$a'' + b'' + \frac{a'}{r} + \frac{b'}{r} + \frac{2{b'}^2}{f} = 16\pi G_0 \hat{p} \tag{73}$$

$$p' = \frac{-2b'}{2}(p + \rho) + \Gamma p \tag{74}$$

in which we have defined $(\hat{\rho}, \hat{p}) = C^4 \frac{e^{a+b/2}}{X+1/X}(\rho, p)$ with $X = C^2 e^{(a+b)/2}$, and the last equation is the hydro-static equilibrium equation involving a new term relative to the GR case with $\Gamma = G'/G$, because of the varying G(r). Notice that we have assumed vacuum on the dark side. Notice also the still constant G_0 in those equations meaning that again the spatial dependence of G is incorporated in densities and pressures. As in GR we can solve the first 3 equations for the unknown a(r), b(r) and $\rho(r)$ (numerically because an analytical treatment only leads to horrible formulas) if we assume a w(r) for the equation of state: $p(r) = w(r)\rho(r)$. And then the last fourth equation becomes a constraint that would not be automatically fulfilled if G was constant. The Γ term again allows to unblock our gravity. However the effect of this term is probably not needed if w(r) is free to vary as should actually be the case in most physical situations. Indeed we have then four unknowns $a(r), b(r), \rho(r)$ and $w(r)$ for our four equations so that the physical equilibrium will naturally select a solution for them without any need for the Γ term. Only in case w(r) itself is not able to vary enough anymore, will the Γ term start to play its role. For instance, this is expected to occur when w(r) saturates at its maximum value 1/3 for an ultra relativistic fluid, such as in a neutron star or a BH. What is reassuring in this analysis is that, thanks to the matter degrees of freedom, in most isotropic situations the blockage does not even occur (a varying G is not needed) as it did in the homogeneous isotropic case. A less symmetric context is expected to be even more favourable as we anticipated earlier. Now remains to be investigated the non static isotropic case to study a star collapse. It might be that the variation of G not only in space but also in time as in cosmology will allow an effective transfer of matter between the two sides.

Before commenting the numerical solutions we can notice that in the weak field approximation in approximate vacuum, treating f as the constant $4\frac{C^4+1}{C^4-1}$ the PPN development of the above solutions brings to light a possible departure from GR at the PostPostNewtonian level since:

$$A_{GR} \approx 1 + 4U + 6U^2 \tag{75}$$

$$B_{GR} \approx 1 - 4U + 8U^2 - 12U^3 \tag{76}$$

$$A_{f\neq 4} \approx 1 + fU + \frac{f(f-1)}{2}U^2 \tag{77}$$

$$B_{f\neq 4} \approx 1 - fU + \frac{f^2}{2}U^2 - f\frac{2+f^2}{6}U^3 \tag{78}$$

This makes clear that for $f \neq 4$ redefining the coupling constant to match GR at the Newtonian level, which amounts to replace U by 4U/f in the above expressions, a discrepancy would remain at the PPN level relative to GR predictions.

$$A_{f\neq 4} \approx 1 + 4U + 8(\frac{f-1}{f})U^2 \tag{79}$$

$$B_{f\neq 4} \approx 1 - 4U + 8U^2 - \frac{32}{3}(\frac{2+f^2}{f^2})U^3 \tag{80}$$

For $4 \leq f = 4\frac{1+1/C^4}{1-1/C^4} \leq \infty$ the departure from GR is the greatest for f infinite (C=1) :

$$A_{DG} \approx 1 + 4U + 8U^2 \tag{81}$$

$$B_{DG} \approx 1 - 4U + 8U^2 - \frac{32}{3}U^3 \tag{82}$$

but should not be detectable as C is much too big.

In the strong field regime we need to rely on numerical approximation methods to understand what's going on near the Schwarzschild radius. The numerical integration in Geogebra (using NRésolEquaDiff) was carried on and the resulting b(r) are shown in Figure 13 for various C values. It is found that as C increases b(r) will closely follow the GR solution near the Schwarzschild radius over an increasing range of b(r) which can be many orders of magnitude and perfectly mimic the GR black hole horizon, however at some point the solution deviates from GR and crosses the Schwarzschild radius without singularity. Therefore, as far as the numerical integration is reliable our theory appears to avoid horizon singularities (true Black Holes) for any finite C and not only C=1. This means that the collapsed

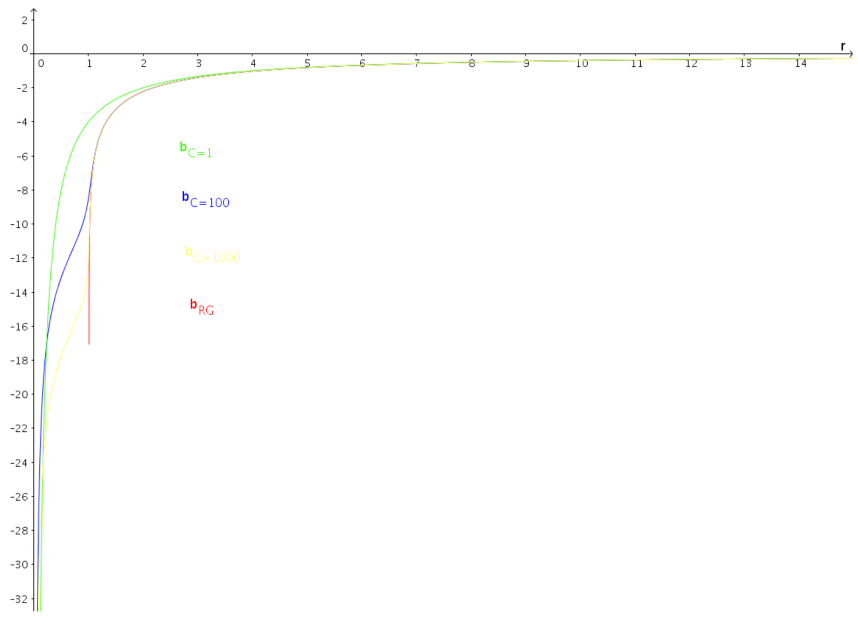

Fig. 13. b(r) near the Schwarzschild radius (r=1) for various C values

star will only behave as a Black Hole for a finite time after which the external observer will be able to learn something about what's going on beyond the pseudo Horizon. Indeed, the resulting object having no true horizon is in principle still able to radiate extremely red-shifted and delayed light or gravitational waves emitted from inside the object. The classical picture of a collapse toward a central singularity could therefore also be probed which is interesting because we can imagine various different original mechanisms to avoid the central singularity.

We actually already have an obvious one, not needing any non trivial extension of our framework. Indeed we remember that all small domains in which the metric remained unchanged at the transition even those around black holes, should eventually naturally shrink and disappear before the end of the cosmological cycle, meaning that these areas will also be in the ocean and therefore loose their gravitational strength as well so that the content of all compact objects, black holes included will be re-dispersed in the universe in such a way that our universe will be re-homogenized at the start of a new cycle. Even if this shrinking takes billion years from the point of view of the outside observer, the observer free falling into the Black hole will see this happening in a very short time and destroying the star before the end of its collapse, therefore avoiding the central singularity.

It's good that we don't need any hypothetical matter transfer mechanism between the two metrics to avoid a central singularity. However it is still tempting to imagine a massive transfer of the star matter to the dark side near the horizon where the g_{00} metric elements are expected to cross each other. Again this process would be extremely fast from the point of view of an observer accompanying the collapse whereas it would take billion years for the far away observer facing an apparently stable black hole.

Having spatial domains and frontiers only required a quite straightforward extension of our framework to relate the limiting values of fields and their derivatives inside each domain as they tend to the common frontier (each domain is closed i.e. does not contain it's boundary). We already did this and it can also describe the energy momentum exchanges between the two domains on both sides of the frontiers. But now we would also need to describe much less trivial relations at frontiers to allow an exchange mechanism to take place between our and the dark side, a program whose practicability remains highly dubious. Yet such mechanism would be interesting as it would open new possibilities: imagine that, for yet obscure reasons, a reset to C=1 could take place somewhere in the inner region of a collapsing star, allowing the two faces of the Janus field to get very close to each other at the center (thanks to C=1 and because this is where the own star potential vanishes) but also producing a huge discontinuous potential barrier between the interior volume and the outside. Then there are observational motivations for exchange mechanisms not only at the center but also at the discontinuous frontier: shocks and matter anti-matter annihilation are expected at the discontinuity (an excess of gamma radiation from our Milky Way giant black hole has indeed been reported [22]) if it is the bridge toward the dark side and it's presumably anti-matter dominated fluid. Further GWs radiation might also be generated which would be much less natural from a regular GR Black Hole [23]. The surface at which the discontinuity is sitting might also behave like the hard shell of a gravastar [45] and would likely produce the same kind of phenomenological signatures such as echoes following BH mergers which might already have been detected [23]. Eventually in the vicinity of stars as well as in "Black Holes" we can't exclude a transfer of matter and radiation that would proceed in the opposite way feeding them and increasing their total energy : a possible new mechanism to explain the unexpectedly high gravific masses of recently discovered BH mergers but also an attractive simple scenario to explain the six SN like enigmatic explosions of the single massive star iPTF14hls if they resulted from a succession of injections of antimatter from the dark side[41]. Such discontinuities in the vicinity of stars could also block matter accumulating in massive and opaque spherical shells around stars : a possible scenario to explain the reduced light signal from the recently discovered neutron stars merger.

Before leaving this subject lets mention that a Kerr type solution for rotating pseudo black holes also remains to be established in our framework which is postponed for some future paper. But it is already clear that both conjugate metrics as well as the Minkowski metric in between them would better be expressed in ellipsoidal coordinates hence in the form given by [46] Eq 21 for the Minkowski metric and Eq 22 or similar for the ensatz in input to our differential equations, and this should significantly ease the computation.

6.2. *Back to Gravitational Waves*

On 17 August 2017, LIGO/Virgo collaboration detected a pulse of gravitational waves,[72] named GW170817, associated with the merger of two neutron stars in an elliptical galaxy 40Mpc from the earth. GW170817 also seemed related to a short (≈ 2 second long) gamma-ray burst, GRB 170817A, first detected 1.7 seconds after the GW merger signal, and a visible light observational event first observed 11 hours afterwards, SSS17a.

The association of GW170817 with GRB 170817A in both space and time is strong evidence that neutron star mergers do create short gamma-ray bursts and that light propagated in this case at the same speed as the gravitational waves within 10^{-15} times the speed of light: 10^{-8} probability to obtain this by chance [73].

If confirmed (no other such coincidence occurred since then, three years later, despite a significant upgrade of the detectors and the detection of many other neutron star merger candidates) the consequence for DG is that light and GW can propagate on the same geodesics over distances as long as 40Mpc. This is expected before the transition redshift because at this epoch our side scale factor dominates by at least $a^2 \propto 10^{20}$ the dark side one so the dark side geometrical terms are suppressed relative to our side terms by $det(g) \propto a^8$ hence at least 80 orders of magnitude. In that case GWs and our side light propagate on almost the same geodesics.

However, following the transition redshift, GW are now supposed to propagate essentially along the geodesics of the dark side metric because now the relative strength of our and the dark side geometrical terms is inverted while, in principle, the light that we can see still propagates on our side. Of course the background being in conformal form on the two sides does not produce any difference for massless waves however the fluctuations i.e. the potentials encountered by light and GW during their propagation are supposed to be opposite: GW see potential hills when light sees potential wells and vice-versa and this alone is expected to produce delays much larger than observed between light and GW, given the typical potentials on the largest scales at the level of 10^{-5}. The effect of our galaxy alone outside a radius of 100kpc would be greater than observed by 10 orders of magnitude.

Then there are only two possible ways to save the theory: either the light received with GRB 170817A, against all odds, mainly propagated on the dark side metric as the GW of GW170817 (first option) or the GW propagated on our side metric as the light of GRB 170817A (second option) just as would have been the case before the transition redshift.

- The first case would imply that a binary neutron star merger into a black hole is able to emit light on the dark side which is not so surprising our pseudo black holes being the perfect places (near the pseudo Horizon or the BH center) for transfers between the two metrics. The fact that this light could be detected on earth, hence on our side, is however much more surprising: if true it would imply that most structures from the dark side are actually visible and detectable and we would expect to be able to see many dark side structures, for instance those situated near the center of our side large scale voids which are expected to be mainly filled by dark side matter. This is difficult to imagine except if for yet unknown reasons, matter on the dark side is essentially in the form of dark matter. This last possibility is however plausible given that in DG, our side and the dark side don't have symmetric roles : the symmetry of the equations is broken by the initial conditions: our side is expanding while the dark side is in contraction (may be eternally) so we have no strong reason to believe that the ratio of normal to dark matter should be the same on the dark side as it is on our side while it remains possible that when radiation on the dark side meets a field discontinuity or a net zero potential producing the conditions for metric crossings (this must occur in between an over-density and an under-density fluctuation and since there the background metrics are actually equivalent from the point of view of a massless particle, in principle a transfer is possible) on it's trajectory, it's transfer to our side will be much favoured relative to the reversed process. So apart from the exceptional case (extreme pressures and gravitational fields) of a neutron star collapse to a Black Hole that would produce the transfer of matter and radiation to the dark side, the normal behaviour of radiation from the dark side meeting a discontinuity could be a transfer to our side. Now since such discontinuities are expected to be localized in the vicinity of the most condensed forms of matter (planets and stars) the light from GRB 170817A which has propagated on the dark side, presumably was

transferred to our side just before reaching us in which case we expect no significant time delay and are motivated to seek for a discontinuity near and around the solar system. It remains that we have no reason to forbid part of the emitted photons to travel also on our side and those may arrive several years later relative to the GWs and photons that propagated on the dark side: this could explain the recently reported observation that, very unexpectedly, the X rays signal from GW170817, now several years later shows an excess increasing with time which is difficult to explain within the current paradigm[74].
Notice that this scenario is obviously only tenable provided there are places at which the two metrics are completely equivalent from the point of view of massless fields so that a transfer mechanism at domain frontiers becomes natural. Only our favoured version of DG (not the $\zeta \propto a^{-4}$ one) allows this as there are always regions in between over-densities and under-densities at which the local potential vanishes so that only the backgrounds can make the difference, yet those backgrounds are completely equivalent for massless particles. For a massive particle on the other hand it is rather the crossing of the g_{00} elements that would make the two metrics equivalent and this is only possible in a very strong gravity regime (near or beyond the Schwarzschild radius of a pseudo Black Hole).

- We may not be in position to completely exclude however the second option meaning that not the whole universe transited at the transition redshift but only a sub-part of it and that regions in which the scale factor was not renormalized allowing light and GW to propagate at the same speed on our side, can extend over as much as 40 Mpcs. The option of a partial transition over a spatial sub-domain is actually unavoidable as it is also actually required to solve another issue that we already identified : if our side had transited over the whole universe, all stars and planets would have lost their gravitational strength and exploded at the transition redshift. It's rather the possibility that such sub-domains could extend over beyond 40 Mpcs distances which is disappointing because then the inside dynamics of smaller structures such as galaxies could not be helped by the dark side. At such smaller scales instead all our predictions would not depart from the LCDM predictions. So the only remaining difference with LCDM for the growth of structures would be in larger structures like voids that presumably define those regions that transited (renormalized their scale factor) while regions in which our side matter dominates, galaxy clusters along filament, did not transit and we would have to assume that this is where the GW and GRB from 170817 propagated. Notice however that our scenario in Fig. 8 makes this plausible because again it is only the mean density that is at play to define the domains. So even a low density region extending over 40 Mpcs could locally remain in a global domain which density dominates the dark side one on the mean.

Therefore, the remaining question for the following sections is whether such sub-domains really need to extend over as much as 40 Mpc (option two) or alternatively (option one) whether we can rely on plenty of small sub-domains about galaxies. In the much more interesting first option (sill trusting the GW-GRB coincidence of august 17 2017) in which almost all the universe transited except small domains about galaxies or even individual stars, the dark side could hopefully help us understand the rotation of galaxies and the MOND empirical law...

In the following we do not decide between the two options to avoid missing any interesting new phenomenology but let's keep in mind that GW and GRB 170817 has far reaching implications for DG and wait and see if this can be confirmed by other similar events.

7. CMB anomalies

Following the recent discovery of a still not physically elucidated CMB new foreground that significantly cools down the temperature ($\approx$ 15 microKelvin) in the direction of nearby spiral galaxy halos[112], a foreground that remarkably correlates with the longstanding issue of the large scale anomalies of the CMB[113], it is not excluded that the Planck cosmological parameters are in error, particularly if the new foreground is not negligible on scales related to the eISW effect. If for instance the density of matter ω_M has been underestimated by Planck because of this foreground then hopefully no correction is actually needed for DG at redshifts just beyond our transition redshift. At the same time a several percent greater matter density would be helpful to explain most of the recently discovered JWST anomalies while subtracting the new foreground should also imply less power on the CMB largest scales probably meaning that LCDM overestimates the Late ISW effect which then favours most alternatives (including DG) that predict less Late ISW.

The new unexpected foreground is challenging all so far considered mechanisms to explain it. As it is frequency independent, we are encouraged to suspect an exotic gravitational effect along the line of sight. However lensing effects or any effect deviating optical rays would not produce a systematic drop of temperature but rather a smoothing of the fluctuations on the corresponding scales. So it appears that we have to deal with a kind of new gravitational effect able to absorb part of the photons irrespective of their frequencies. This really looks like a confirmation of the ability of photons within DG to transit between the two sides of the universe anywhere the local potential vanishes and the regions surrounding small scale density fluctuations such as galaxies are naturally where we expect to find these vanishing potentials more often. So the newly discovered anomaly adds support to our first scenario to explain the GW170817 and GRB 170817A arrival coincidence: here photons and GW apparently really propagated on the dark side. What makes this scenario also appealing is that we don't need to explain large regions extending over more than 40 Mpcs in which the scale factors transition did not take place. So we don't need to rely on a scenario with drifting frontiers and shrinking domains as in Fig. 8 which was also necessary to get a cosmological density increase as the universe approached the transition redshift. Instead we can imagine that the island and cosmological domains have always existed since the bigbang, the cosmological domain was from the begining a domain of smoothly distributed Dark matter while the islands were the domain of a condensed form of DM, may be black holes from a previous cycle of the DG universe.

8. Asymptotically static domains

In this section we are specifically interested in the island domains that were not submitted to the renormalization of the scale factors. The most natural assumption is that the background metric of such small finite domains is completely frozen in a perfectly static state, all the more since, as we shall soon see, this is amazingly required by the most obvious interpretation of the Pioneer effect.

We then have two kind of spatial domains. The cosmological evolving one (the ocean) and plenty of finite small frozen ones (the islands) for which an homogeneous evolving background would not make much sense. On the other hand for the cosmologically evolving domain around the islands (the ocean) including a priori unbounded scales, a cosmological metric of course still makes sense.

In the islands, the metrics are therefore asymptotically Minkowskian but rather in standard cosmological time coordinate (hence the expansion effects are switched off in such domains while their clock rates are still not drifting with respect to clocks in the evolving domain). So high density regions, for instance about stars, are understood to be cut-out of the rest of the expanding universe, implying a discontinuity at their frontier surface defining a new volume which is not anymore submitted to the expanding:

$$d\tau^2 = a^2(t)(dt^2 - d\sigma^2) = dt'^2 - a'^2(t')d\sigma^2 \tag{83}$$

cosmological metric ($d\sigma^2 = dx^2 + dy^2 + dz^2$), but to the new Minkowski metric:

$$d\tau^2 = a^2(t)dt^2 - C_{frozen}^2 d\sigma^2 = dt'^2 - C_{frozen}^2 d\sigma^2 \tag{84}$$

where C_{frozen} stands for the reached value of the scale factor at the time it froze. Again, this is very natural first because such finite bounded domains are extremely in-homogeneous so the very idea of an homogeneous background would not make any sense in them so that it is instead reasonable to treat them as asymptotically Minkowskian. Second because for a very non linear fluctuation, for instance a galaxy halo, in GR as well background expansion effects are negligible.

What is then crucial for us is that the domain of validity of the evolving background solutions according (83) has frontiers which could play a role for matter and radiation transfers between our and the dark side (and not only at BH pseudo-horizons where at least the g_{00} elements of the conjugate metrics cross each other) as needed for instance to let the light from GW170817 travel on the dark side metric and still be able to reach our detectors well synchronized with the GW signal.

Moreover everything carrying energy-momentum crossing the frontiers of the evolving cosmological background domain on our side (resp on the conjugate side) can increase or decrease the energy density in the cosmological domain, an effect which may be also needed to improve our fit to H(z) data.

The remaining question now is : which ones are the actual energy-masses that must have flipped to the 1/C domain at the transition redshift resulting in switching off almost all the density of our side of the universe in the cosmological equation.

As we shall see later the weak lensing data that we have at redshift less than z_{tr} imply the existence of non linear structures as large as galaxy halos while the dark side structure corresponding to galaxies and galaxy clusters are merely voids which have limited density contrast and cannot produce enough lensing at this time.

So we are led to conclude that at least at redshifts closely following the transition the cosmological domain should extend to everything outside halos and our static domains have their frontiers delimiting halos. But the situation could have evolved quite much given that the mean density of our side is now smaller by a factor $\approx (1 + z_{tr})^6 \approx 1.7^6 = 24$ compared to the dark side one so that we expect lensing effects to receive a greater contribution from the dark side voids around them which are now deep voids in a much denser background than on our side. Static domains might also have shrunk and be much smaller now.

Anyway, we see that from the transition redshift to now the gravific masses at work which effects we can probe in the universe are the fluctuations on the dark side (of type M_4) (we shall see in a next section that a void in that distribution can perfectly mimic a halo of dark matter on our side), but also the condensed forms of matter on our side (of type M_1) : stars, planets or inner part of galaxies.

Eventually static domains could play several key roles at the same time:

- They provide frontiers may be allowing to understand matter radiation exchange between our and the dark sector but also between the finite bounded static domains and the rest of the universe with a possible effect on H(z).
- The static domains can remain C-domains on our side rather than 1/C domains insuring that their masses are still gravific. Even though those domains were not renormalized from C to 1/C at transition redshift, their clocks need to remain synchronized with the evolving domains background clocks driven by a scale factor in the accelerated expansion regime. This is actually needed for our reference clocks which happen to be in the static domains to allow us to see the universe expansion accelerated by comparing the frequencies of cosmological photons to these reference clocks frequencies. In the next sections we shall deal with this issue and explain how all clocks can remain synchronized.
- We might not only need the equality of densities but also the equality of pressures from both sides of the Janus field to trigger the transition to acceleration. It is not granted that those two conditions can be met simultaneously and exactly between our and the dark side in the cosmological

domain even though we expect pressures to be similar when the densities are equal. We have identified two other mechanisms that could help, if needed, get pressure and density equality at the same time between the cosmological domains:

- Neutrinos getting massive at low redshifts (the ultracool transition we discussed earlier) with a direct effect on the relative fraction of fluids in the relativistic and non relativistic regime.
- Quantum mechanics if the only contributors to the cosmological evolving domain are those particle wave functions that are dispersed rather than in their collapsed state. Indeed any object less than 1 micron (except may be a PBH) in the very rarefied intergalactic medium has a decoherence time more than 1 second (and more than 10 days for 0.1 micron particles) so that it's mass energy (we are following a realistic interpretation of QM) is most often diluted in a large volume insuring it should not represent a large fluctuation from the mean universe density which order of magnitude is atoms per cube meter. So most of the diffuse matter-energy in the form of gas and dark matter far from condensed structures should actually be in this un-collapsed state and would not produce frozen regions at the contrary to the collapsed forms of matter. Then any variation of the fundamental collapse triggering parameter should result in an increase or decrease of the fraction of energy matter in the evolving domain rather than in the static domains and then result in a contribution to the total density vs pressure of the cosmological domain. Eventually we are led to the fascinating idea that the physics of the QM wave function collapse could play a role at the cosmological level.

Anyway, only the highly clustered forms of matter e.g. stars, planets, micro PBHs and may be up to even dust particles of a sufficient size should be able to generate their own static domain of the scale factor evolution in their vicinity in which these can remain in the frozen regime described by (84). In the two following sections we shall explore all the consequences and new predictions related to static domains among which the Pioneer effect as a natural outcome. A Pioneer effect, as we shall explain, both qualitatively and quantitatively is predicted to be observable if we can compare the rate of two clocks on both sides of the frontier of a static domain if in such domain the Minkowski background metric synchronizes on the cosmological metric of the contracting dark side rather than on the cosmological metric of our expanding side. In the two following sections we propose an involved mechanism based on an extended extreme action principle that would allow a Pioneer effect to occur periodically with a period of $\approx 10^5$ years in such a way that the agreement with all other cosmological observable would be preserved on the mean. To not spoil this agreement one could alternatively avoid all this complexity just by admitting that the Pioneer effect is a rare and ephemeral event, therefore negligible on cosmological scales, or even better, if it is not so rare and ephemeral that it could be a natural explanation for wiggles observed in the H(z) expansion history. In the same vein, we may also not exclude the possibility that static domains can momentarily completely de-synchronize their background Minkowski metric in which case we might have transient regime of zero $\ddot{a}(t)$ in (84) resulting in constant H(z)/(1+z) which would help obtain the H(z) increase on the mean that we found necessary for a good fit to Planck Power spectra.

The next sections are therefore a bit more speculative and the reader not interested in the Pioneer effect or in the unavoidable complication to get a cyclic Pioneer effect can jump directly to section 13.

9. The physics of static domains

Because we want to understand the Pioneer anomaly, and for several other reasons discussed earlier we are led to seriously consider that the static domains introduced in a previous section are real. These obviously require new synchronization mechanisms between clocks from the static and evolving background domains which we shall detail now. In subsequent sections we shall focus on some of the very rich phenomenological related outcomes.

9.1. *Actions and space-time domains*

In a space-time domain D_{int} cut out from the expanding rest of the universe D_{ext} we still have as usual the Einstein Hilbert (EH) action for the asymptotically Minkowskian Janus Field $g^{\eta}_{\mu\nu}$ added to SM actions for F and $\tilde{F}$ type fields respectively minimally coupled to $g^{\eta}_{\mu\nu}$ and $\tilde{g}^{\eta}_{\mu\nu}$ (the superscript here does not mean that the two sides of the Janus field are asymptotically identical but merely both asymptotically flat and static). However we may add to such action, an independent Einstein Hilbert action for a pure scalar-η homogeneous and isotropic Janus field which we write $a^2_{int}\eta$. The purpose of this action is just to extend to D_{int} the effect of the background which dynamics was determined by extremizing the D_{ext} action and solving the implied equations for the FRW ensatz to get the external scale factor evolution $a_{ext}(t)$. In other words in the D_{int} action for the scalar-η field the scalar field is not dynamical but it's evolution is driven by the external background field. Indeed to insure the synchronization of interior and exterior clocks we postulate that the Hubble rates H_{int} and H_{ext} are still equal implying that $a_{int} = C^2 a_{ext}$ just because only the exterior scale factor was renormalized by $1/C^2$

at the transition redshift. Then the total action in D_{int} is [j]:

$$\int_{D_{int}} d^4x(\sqrt{g}R + \sqrt{\tilde{g}}\tilde{R})_{g=a_{int}^2\eta} + \tag{85}$$

$$\int_{D_{int}} d^4x(\sqrt{g}(R+L) + \sqrt{\tilde{g}}(\tilde{R}+\tilde{L}))_{g^\eta} \tag{86}$$

The advantage of adding a separate action for an independent non dynamical $\eta - scalar$ field in D_{int} is not clear at this level because there is no shared field between the two kinds of actions. The point is that g^η is not only determined by its equations of motion. It could be asymptotically identical to any Minkowskian metric, for instance any of the form :

$$d\tau^2 = f^2(t)dt^2 - C^2 d\sigma^2 \tag{87}$$

in which the f(t) function is of course pure Gauge inside D_{int} however it is needed to determine how clocks within D_{int} may actually drift in time with respect to clocks in D_{ext}. Since f(t) is free as of now our purpose is indeed to introduce an additional driving mechanism relating f(t) to $a_{int} = C^2 a_{ext}$. We could just postulate these are equal again to prevent the local clocks in D_{int} to drift with respect to D_{ext} clocks, however we are interested in a more involved mechanism actually allowing such drifts to occur at least momentarily as this is needed to produce Pioneer like effects. Our total action will be helpful just to later introduce such mechanism and establish a somewhat less trivial connection between f(t) and $a_{int}(t)$ in D_{int}.

In our approach the background metric is purely Minkowskian in the solar system while in GR there are expansion effects only significant on scales beyond those of galaxy clusters and almost completely negligible but not strictly vanishing in the solar system.

9.2. *Field discontinuities*

If the mechanism which translates the $a_{int}(t)$ evolution into f(t) evolution is momentarily switched off, we expect a field discontinuity for the g_{00} metric element at the frontier between a momentarily stationary scale factor domain D_{int} and evolving outside D_{ext} domain.

Let's stress that those new kind of discontinuities are not related at all to our permutation symmetry and the related discrete cosmological transition process that could trigger the acceleration of the universe. Now the usual conservation equations for matter or radiation apply when crossing such frontiers though in presence of genuine potential discontinuities. Indeed it's possible to describe the propagation of the

[j] There is may be one alternative possible way to obtain a background metric in D_{int} in a fully dynamical way by adding source terms which densities would be averages over $D_{int} + D_{ext}$. Then the implied equations of motion for a dust universe, $\rho_{[D_{int}+D_{ext}]}/a^3_{[D_{int}+D_{ext}]}$=Const could still be compatible with $\rho_{D_{ext}}/a^3_{D_{ext}}$=Const, the scale factors $a_{[D_{int}+D_{ext}]}$ and $a_{[D_{ext}]}$ evolution being slightly different.

wave function of any particle crossing this new kind of discontinuous gravitational potential frontier just as the Schrodinger equation can be solved exactly in presence of a squared potential well : we just need to require the continuity of the matter and radiation fields and continuity of their derivatives at such gravitational discontinuity. Since the differential equations are valid everywhere except at the discontinuity itself where they are just complemented by the former matching rules we obviously avoid the nuisance of any infinite potential gradients and eventually only potential differences between both sides of such discontinuity will physically matter. For instance we can now have have $(\rho a^3)_{before-crossing} = (\rho a^3)_{after-crossing}$ in contrast to what we had following the permutation transition $(\rho_{before-crossing} = \rho_{after-crossing})$.

9.3. *Space-time domains and the Pioneer effect*

The following question therefore arises: suppose we have two identical clocks exchanging electromagnetic signals between one domain submitted to the expanding $a_{int}(t)$ and another without such effect. The reader is invited to visit the detailed analysis in our previous publication [15] starting at page 71. We shall only remind here the main results. Electromagnetic periods and wavelengths are not impacted in any way during the propagation of electromagnetic waves even when crossing the inter-domain frontier. Through the exchange of electromagnetic signals, the period of the clock decreasing as $a(t)$ can then directly be tracked and compared to the static clock period and should be seen accelerated with respect to it at a rate equal to the Hubble rate H_0. Such clock acceleration effect indeed suddenly appeared in the radio-wave signal received from the Pioneer space-crafts but with the wrong magnitude by a factor two: $\frac{\dot{f}_P}{f_E} \approx 2H_0$ where f_P and f_E stand for Pioneer and earth clocks frequencies respectively. This is the so called Pioneer anomaly [12][13]. The interpretation of the sudden onset of the Pioneer anomaly just after Saturn encounter would be straightforward if this is where the spacecraft crossed the frontier between the two regions. The region not submitted to $a_{int}(t)$ (at least temporarily) would therefore be the inner part of the solar system where we find our earth clocks and where indeed various precision tests have shown that expansion or contraction effects on orbital periods are excluded during the last decades. Only the origin of the factor 2 discrepancy between theory and observation remains to be elucidated in the following sections as well as a PLL issue we need to clarify first.

9.4. *Back to PLL issues*

As we started to explain in our previous article [15] in principle a Pioneer spacecraft should behave as a mere mirror for radio waves even though it includes a frequency multiplier. This is because its re-emitted radio wave is phase locked to the received wave so one should not be sensitive to the own free speed of the Pioneer clock.

Our interpretation of the Pioneer effect thus requires that there was a failure of on board PLLs (Phase Lock Loop) to specifically "follow" a Pioneer like drift in time or even a failure that forced the analysis of the data in open loop mode. As for the first hypothesis, we already pointed out that nobody knows how the scale factor actually varies on short time scales: in [15] we already imagined that it might only vary on very rare and short time slots but with a much bigger instantaneous Hubble factor than the average Hubble rate. This behaviour would produce high frequency components in the spectrum which might have not passed a low pass filter in the on board PLL system, resulting in the on board clocks not being able to follow those sudden drifts. The on board clocks would only efficiently follow the slow frequency variations allowing Doppler tracking of the spacecrafts. Only when the integrated total drift of the phase due to the cumulative effect of many successive clock fast accelerations would reach a too high level for the system, this system would "notice" that something went wrong, perhaps resulting in instabilities and loss of lock at regular intervals [15]. This view would be even better supported if our clocks and rods are submitted to the scale factor evolution not continuously but rather through the succession of discontinuous steps we considered earlier. The failure of the PLL system is then even better understood for discontinuous variations of the Pioneer clock frequency with respect to the earth clock frequency. As a result, the frequency of the re-emitted wave is impacted by the Pioneer clock successive drifts and the earth system could detect this as a Pioneer anomaly.

9.5. *Cyclic expanding and static regimes*

We are now ready to address the factor two discrepancy between our prediction and the observed Pioneer clock acceleration rate. We know from cosmology that, still in the same coordinate system, earth clocks must have been accelerating at a rate H_0 with respect to still standing electromagnetic periods of photons reaching us after travelling across cosmological distances (thus mainly in D_{ext}): this is nothing but the description of the so called cosmological redshift in conformal time rather than usual standard time coordinate.

On the other hand the Pioneer effect itself requires that not all regions have their clocks submitted to the same scale factor at the same time but some regions instead have their clocks drifting at rate $2H_0$ with respect to those from other regions.

This seems to imply that through cosmological times, not only earth clocks but also all other clocks in the universe, may have spent exactly half of the time in the $2H_0$ regime and half of the time in the static regime, in a cyclic way. It would follow that the instantaneous expansion rate $2H_0$ as deduced from the Pioneer effect is twice bigger than the average expansion rate (the average of $2H_0$ and zero respectively in the expanding and static halves of the cycle) as measured through a cumulative redshift over billions of years.

In our previous article we presented a very different more complicated and less natural explanation on how we could get the needed factor two which we do not support anymore. This article also discussed the expected field discontinuities at the frontier between regions with different expansion regimes, and likely related effects which we still support. Those discontinuities do not necessarily imply huge potential barriers even though the scale factors have varied by many orders of magnitude between the Big Bang and now. At the contrary they could be so small to have remained unnoticed as far as our cycle is short enough to prevent some regions to accumulate a too much drift relative to others. We are now at last ready, having introduced the main ideas, to detail the mechanism relating $f(t)$ to $a_{int}(t)$ in a D_{int} domain.

10. Driving mechanism for frozen domains and frontier dynamics

10.1. *A sophisticated periodic mechanism*

- First postulate : A D_{int} domain has a new own non dynamical Minkowski metric in addition to the DG fundamental non dynamical Minkowski metric from which we built the Janus field which is still there in both D_{int} and D_{ext}. This new metric is just (84):
$$d\tau^2 = a_{int}^2(t)dt^2 - C_{frozen}^2 d\sigma^2 = dt'^2 - C_{frozen}^2 d\sigma^2 \tag{88}$$
while the old non dynamical Minkowski metric is still :
$$d\tau^2 = dt^2 - d\sigma^2 \tag{89}$$
Obviously the dynamics of the background in D_{ext} (the scale factor $a_{ext}(t)$) is what determines the new non dynamical metric.
- Second postulate: The dynamical metric in D_{int} is asymptotically successively:
$$d\tau^2 = D_{frozen}^2 dt^2 - C_{frozen}^2 d\sigma^2 \tag{90}$$
which is completely frozen and:
$$d\tau^2 = \frac{a_{int}^4(t)}{D_{frozen}^2} dt^2 - C_{frozen}^2 d\sigma^2 \tag{91}$$
in which clocks are found drifting at the double rate $2H_0$. D_{frozen} in (91) stands for the last frozen value of $a_{int}(t)$ at the time the metric switched from (90) to (91). Of course D_{frozen} has a new value at each cycle.
Therefore, in D_{int} we have an alternate cyclic succession of what would seam to be the two sides of a new emergent Janus field about (88) except that at any time only one physically shows up and only as an asymptotic value of the D_{int} dynamical field.
As we already noticed earlier the asymptotic behaviour is not determined by field equations in D_{int} and as promised our postulates provide the needed constraints according to which $a_{ext}(t)$ from D_{ext} drives this asymptotic behaviour in D_{int}.
The cyclic succession of (90) and (91) makes the D_{int} dynamical field asymptotically evolve as (88) on cosmological times but this is a mean.
Of course the fact that metrics (90) and (91) look like the two sides of a new D_{int} Janus field about (88) is not an accident. Presumably the existence

of (91) is just the consequence of the existence of the other side (90) and (88) in between. In other words we have a kind of baby universe in D_{int} which background is not (may be not yet) able to evolve by itself but which evolution is completely dictated by D_{ext} according our postulates. Presumably the baby universe will eventually acquire it's full autonomy when the two sides really become the two sides of a genuine new dynamical Janus field starting it's own evolution according it's own action and derived field equations.

- Third postulate : In general the dynamical field is not necessarily asymptotically (90) or (91) in the whole domain D_{int}. Rather half of the time D_{int} is in the static regime and the other half of the time the domain progressively passes in the double rate regime: when this occurs there is a domain frontier that scans the whole D_{int}: upstream (not yet reached area of) this propagating frontier we are still in the static regime while downstream all clocks have been synchronized and are in the double rate regime. At the end of the scan the whole D_{int} is frozen again in the static regime for the next half cycle.
  To describe this the action in D_{int} is the one we have already written in (85) and (86) which we can rewrite now only retaining the double rate regime area in D_{int} and the geometrical terms (the matter actions and static regime area play no role in the following so we drop them out hereafter just for the sake of conciseness):
  $$\int_{D_{int:2H_0}} d^4x(\sqrt{g}R + \sqrt{\tilde{g}}\tilde{R})_{g=a_{int}^2\eta} + \tag{92}$$
  $$\int_{D_{int:2H_0}} d^4x(\sqrt{g}R + \sqrt{\tilde{g}}\tilde{R})_{g^\eta} \tag{93}$$
  Our third postulate is to require this action to be extremum i.e. stationary under any infinitesimal displacement of the hypersurface defined by the frontier of this action validity domain $D_{int:2H_0}$.

Our purpose is to understand the physics that governs the location of the frontier surface of $D_{int:2H_0}$ at any time. Of course determining it will at the same time determine the frontier of the complementary $D_{int:static}$ area. If such surface is moving it will of course scan a space-time volume as time is running out. Having extended the extremum action principle thanks to the third postulate allows to determine this hypersurface.

Indeed the arbitrarily displaced hypersurface might only differ from the original one near some arbitrary point, so that requiring the action variation to vanish actually implies that the total integrand should vanish at this point and therefore anywhere on the hypersurface. Eventually, anywhere and at any time at the domain boundary we have:

$$(\sqrt{g}R + \sqrt{\tilde{g}}\tilde{R})_{g=a^2\eta} + (\sqrt{g}R + \sqrt{\tilde{g}}\tilde{R})_{g^\eta} = 0 \tag{94}$$

This equation is merely a constraint relating the Janus field gravity (terms 3 and 4) to the non dynamical metric (terms 1 and 2) at the hyper surface. Here and from now on we shall omit the "int" subscript for the scale factor unless otherwise specified. Now provided one scale factor dominates the other side one we have:

$$(\sqrt{g}R + \sqrt{\tilde{g}}\tilde{R})_{g=a^2\eta} \approx \pm^{a>>\tilde{a}}_{a<<\tilde{a}}(\sqrt{g}R - \sqrt{\tilde{g}}\tilde{R})_{g=a^2\eta} \tag{95}$$

and then we can make use of the contracted equation 4 to replace:

$$(\sqrt{g}R + \sqrt{\tilde{g}}\tilde{R})_{g=a^2\eta} \approx \pm^{a>>\tilde{a}}_{a<<\tilde{a}}8\pi G(\sqrt{g}T - \sqrt{\tilde{g}}\tilde{T})_{g=a^2\eta} \tag{96}$$

in equation(94) and we can do the same for the g^η part provided $C(t) = a^2(t)/D_{frozen}$ and D_{frozen} dominate their inverse (the common order of magnitude of $C(t)$ and D_{frozen} is simply named C hereafter). Then equation (94) becomes:

$$\pm^{a>>\tilde{a}}_{a<<\tilde{a}}(a^4 < \rho - 3p >_{ext} - \tilde{a}^4 < \tilde{\rho} - 3\tilde{p} >_{ext}) \tag{97}$$

$$\pm^{C>>\tilde{C}}_{C<<\tilde{C}}(C(t)\ D^3_{frozen}F(r)(\rho - 3p) - \tilde{C}(t)\tilde{D}^3_{frozen}\tilde{F}(r)(\tilde{\rho} - 3\tilde{p})) = 0 \tag{98}$$

The $F(r) = e^{2\Phi(r)}$ and $\tilde{F}(r) = e^{-2\Phi(r)}$ here account for the effect of a local assumed static isotropic gravitational potential $\Phi(r)$. The $<>_{ext}$ denote averages over D_{ext}. First and third terms involve a factor which currently has approximately the same magnitude as $a(t)$ in our cold side of the universe (even though third term is actually momentarily evolving at twice the rate of a hence rather as a^2) while second and fourth terms involve a factor which currently has approximately the same magnitude as $\tilde{a}(t)$ (even though fourth term is actually momentarily evolving at twice the rate of $\tilde{a}(t)$ hence rather as $\tilde{a}^2(t)$) if the dark side is also in a cold matter dominated era.

The relative magnitudes of the local densities can be very different from the relative magnitudes of the averages $<>$ given the extremely non linear structures in the current universe. Is this enough to make the relative magnitudes of terms 1 and 2 in the opposite way to the relative magnitudes of terms 3 and 4 ? Unlikely at first sight given the huge expected current ratio $a(t)/\tilde{a(t)} \approx C(t)/\tilde{C(t)} \approx z^2_{crossing} >> 10^{18}$, if $z_{crossing}$ is the redshift of the conjugate scale factors equality probably much greater than the BBN redshift. Then as term 3 >> term 4, just as term 1 >> term 2 the equation today (with negligible pressures) simplifies to :

$$a^4 < \rho >_{ext} + C(t)D^3_{frozen}F(r)\rho = 0 \tag{99}$$

Such equation is satisfactory because the two terms don't evolve in the same way as a function of time: the first and second terms imply clocks drifting at rate H_0 and $2H_0$ respectively. So this can lead us to a trajectory r(t) for our hypersurface. Therefore, for instance in the external gravity of a massive spherical body, planet or star on our side, which radial a-dimensional potential is $\Phi(r) = -GM/rc^2$ and a quite uniform $\rho(r)$ so we may neglect it's radial dependency (for instance in the empty space surrounding a star), and using the fact that $C(t)$ momentarily evolves as $a^2(t)$ we are led to:

$$a(t) \propto e^{\frac{2MG}{rc^2}} \tag{100}$$

This equation gives us nothing but the "trajectory" r(t) of the hypersurface we were looking for. Here obtained in the conformal time t coordinate system, it is also valid in standard time t' coordinate since the standard scale factor and the "conformal scale factor" are related by $a(t) = a'(t')$. It is valid to PN order being understood that the exponential metric is here used for simplicity as a weak field PN approximation of a GR Schwarzschild solution rather than really the DG exponential Schwarzschild solution. This equation I=J implies $\dot{I}/I = \dot{J}/J$ so that:

$$H_0 = -2\frac{d\Phi}{dr}\frac{dr}{dt} \tag{101}$$

From this we learn that the frontier between the two domains is drifting at speed:

$$\frac{dr}{dt} = -\frac{1}{2}\frac{H_0}{[\frac{d\Phi(r)}{dr}]} \tag{102}$$

and therefore could involve a characteristic period, the time needed for the scale factor to scan $e^{\frac{2MG}{r}}$ from the asymptotic value to the deepest level of the potential at which point a new scan cycle is started. Thus we are able to understand both the Pioneer effect when we compare clocks in $D_{int:2H_0}$ and in $D_{int:static}$ but also the average H_0 expansion rate of the universe. Video of an animation is available at [[17]].

We may estimate an order of magnitude of the characteristic period of this cyclic drift assuming that the cycle is over when the frontier reaches the deepest potential levels. For collapsed stars such as white dwarfs or neutron stars this would give a far too long cycle exceeding billions of years because their surface potential is so deep and even much worse for black holes. But the majority of stars have very similar surface potentials even though there is a large variability in their masses and sizes. So we may take the value of our sun a-dimensional surface potential which is about 2.10^{-6} as indicative of a mean and common value. To that number we should add the potential in the gravitational field of the Milky Way and the potential to which the local cluster of galaxies is subjected. Knowing the velocities: 220 km/s of the sun about the center of the galaxy and 600km/s of the local cluster vs the CMB, the virial approximation formula $\frac{v^2}{c^2} \approx GM/rc^2$ may lead us to a crude estimation of each contribution and a total potential near 6.10^{-6}. Then the order of magnitude of the cycle period would be in between 10^4 and 10^5 years.

10.2. *Alternative: a trivial but exceptional mechanism*

Of course the Pioneer effect could be a rare and exceptional event and in this case we could account for it in the most trivial way, just arguing that exceptionally and for yet not clear reasons in some static bounded domains clock frequencies may momentarily evolve (lock their Hubble rate) according the contracting side laws instead of other clocks evolving according the laws of the expanding side, and of course in this case it is trivial to get a $2H_0$ drifting rate between such two kinds of clocks.

11. Other predictions related to frozen metrics

The metrics of (90) and (91) lead to likely testable new phenomenological outcomes. If, as we already pointed out, those are alternating at a high frequency cycle, the g_{00} element mean evolution is almost the same as within GR with short-lived transient deviations that should remain small.

A remarkable exception could occur in the vicinity of compact star surfaces (white dwarfs, neutron stars or our pseudo Black Holes) because it takes much longer time for the scale factor to scan such star strong gravitational potentials up to the star surface. So for instance the asymptotically evolving according (91) and stationary according (90) regions on either sides of the drifting frontier can accumulate an extremely large relative drift of their g_{00} metric elements relative to each other over such a long time, but also a very large drift with respect to the g_{00} metric elements of the D_{ext} region evolving as $a^2(t)$.

This would not only result in much larger discontinuous barriers, able to block or instantaneously accelerate matter, but also large accumulating gravitational redshifts of regions submitted to (90) relative to the external universe. Eventually any kind of radiation emitted from within such region is going to be red-shifted as usual along it's cosmological path to the observer implying an "emission" redshift z_e. However the total redshift should also receive an additional very significant contribution due to the source itself being already shifted if it remained frozen for billion years relative to earth clocks before the emission (we are still reasoning in the conformal time coordinate system) and this should extend the total to the freeze redshift z_{fr}. Now the luminosity distance to BH mergers should be given by $d_L = (1+z_{fr})a_0 r_1(z_e)$. Using the usual $d_L(z)$ formula ignoring that there are actually two different redshifts entering it, the deduced z from the luminosity distance is then in between z_{fr} and z_e and seriously systematically underestimates the physically relevant z_{fr} resulting in overestimating the mergers chirp mass: this is analogous to the argument in [51] except that we don't need lensing and magnification for that in our case. So similarly the true BH masses may remain in the 10 - 12 solar masses range.

Recently a team [101][102] has claimed to have discovered a kind of cosmological coupling of supermassive black holes. In a population of very quiescent elliptical galaxies for which we expect no accretion nor merging activity over cosmological times and therefore no significant variation of the total stellar mass nor the central supermassive black hole mass, the team finds by comparing such population at high and low redshift that the stellar mass indeed does not vary whereas the supermassive

Black Hole mass unexpectedly grows very much and apparently according the simple law $M_{SMBH} \propto a(t)^{3.11+1.19-1.33}$. It is tempting to interpret the $a^2(t)$ redshift of the g_{00} metric element between the region occupied by the SMBH and the outside world really as a mass shift $M_{SMBH} \propto a^2(t)$ from the exterior observer viewpoint, a predicted effect thus less than one sigma from the observed one. In GR indeed mass scales as $a(t)$ in the conformal coordinate system (see [50] p21) but of course the effect is then not detectable because is compensated by reference clocks and rods behaviour whereas it should be really physical in our case from the outside observer viewpoint. However the understanding that we have developed so far does not allow the external gravitation field to be sourced by this variable mass: The gravitational field is understood to be produced by the source inside the zone delimited by the discontinuity and then propagates to the exterior with still the gravitational field of a constant mass (another option would have been to consider that a new gravitational field is produced by the frontier behaving as a new source from the exterior world point of view which could be a variable mass source).

This is however a good opportunity to investigate the possibility to explain a mass growth of BH whether or not coupled to the evolution of the scale factor in our framework. Anyway if such object is isotropic and in quasi vacuum it would appear that having its mass thus its gravitational field varying in time from the exterior world point of view contradicts the Birkhoff theorem which is still valid in DG. So either the vacuum or isotropy condition must be broken. The more realistic possibility is the automatic breaking of the isotropy condition if the frame associated to the Black Hole region encompassed by the discontinuity is rotating with respect to the external world inertial frame. Then an energy exchange is made possible through gravitational wave radiation but then those GWs should carry a negative energy to account for the cosmic increase of the BH mass from the exterior world point of view. This in turn implies that the GWs are emitted in a region in which the conjugate scale factor dominates our side scale factor.

Then the total population of such Black Holes which number density $N_{SMBH} \propto a^{-3}(t)$ if these are located in the cosmological domain dominated by our side scale factor, should then behave as a cosmological fluid with density $\rho_{SMBH}(t) = N_{SMBH}M_{SMBH} \propto a^n(t)$ with n greater than -3 and might naively produce the needed increase of $H(z)$ as z decreases and approaches the transition redshift, corresponding to the Ad Hoc correction that we earlier introduced to better fit $H(z)$ data, except that such BH are actually not understood to be anymore in the cosmological region in which our side scale factor dominates and therefore should no longer contribute to our side cosmological density which is driving the cosmic evolution before z_{tr}. The mass growth of such BH is rather more natural after the transition redshift, because then the cosmological domain is indeed dominated by the conjugate scale factor and a BH in such environment may indeed radiate negative energy waves.

To allow H(z) to increase unexpectedly in the decelerating universe as we approach the transition redshift we therefore still favour our best option: admit that the cosmological domain being open to energy exchanges with other domains (the islands), the energy conservation is violated in this domain: in particular if the cosmological domain is being progressively reduced as we approach z_{tr}, loosing the areas in which the density is the smallest (corresponding to large universe voids) this has the effect of increasing it's mean density (thus H(z)) in the remaining volume. In this scenario, the transition between decelerating and accelerating universe is going to be smoothed: it might be that no more discontinuity in the derivative of H(z) should be detectable.

We also have a discontinuity for g_{ii} metric elements because of frozen C_{frozen} and this could be responsible for a different kind of effects: Shapiro delay or deflection of photons crossing the discontinuous potential. Because D_{int} evolves as (88) on the mean, there is a potentially cumulative hence large effect on cosmological times. On the other hand if the metric in D_{ext} is just as within GR the result of a non linear non trivial superposition of background and local gravity, the effects of the expansion are expected to be highly suppressed if we are not very far away from the sun which is also almost equivalent to a frozen scale factor. So the effect when crossing the discontinuous frontier might remain small though this remains to be investigated in more details!

In particular, it will prove interesting to check whether the implied distortions could actually explain the CMB low multipole anomalies [[60]][[59]], for instance the low quadrupole power and correlations with the ecliptic and galactic planes, and more specifically the order of magnitude of g_{ii} discontinuities related to the presence of the sun (but not anymore necessarily constrained to be at the level of the sun a-dimensional surface potential which is 10^{-6}) needed to get such effect from light rays being deviated according to the Descartes refraction law with effective gravitational indices given by differing g_{ii} on both sides of the frontier. This also obviously requires the frontier surface to not look isotropic from the Planck experiment view point which indeed is not centered at the sun.

Near a BH such discontinuities could be much larger not only implying refraction but also a significant reflection if the effective gravitational optical indices actually differ by a large amount. The question remains opened whether this could help produce echoes of a gravitational wave signal.

12. The MOND phenomenology

As already pointed out DG crucially differs from GR in the way global expansion and local gravity work together. Any anomaly in the local physics of the solar system or galaxy seemingly pointing to effects related to the Hubble rate is completely puzzling in the context of GR while it may be naturally explained within DG. Not only the Pioneer effect but also MOND phenomenology seem related to the H_0 value.

We derived in a former section the speed $\frac{dr}{dt} = -\frac{1}{2}\frac{H_0}{d\Phi(r)/dr}$ at which a frontier sitting at an isopotential between internal and external regions should radially propagate in the potential well of a given body. From this formula the speed of light $\frac{dr}{dt} = c$ is reached anywhere the acceleration of gravity equals $cH_0/2$. This appears to be the order of magnitude of the MOND acceleration and the corresponding radius even closer to the MOND radius beyond which gravity starts to be anomalous in galaxies [20][28]. Also remember that we assumed a radially uniform fluctuation to derive the speed formula for our hypersurface which amounts to consider that $d\Phi(r)/dr$ is its leading contribution so such estimation can only be very approximate. We are therefore tempted to suspect that something must be happening near the MOND radius due to frontier discontinuities propagating (and dragging matter) at a speed approaching the speed of light. Our best guess is that this is the radius beyond which the cosmological non Minkowskian background metric takes over.

Another kind of argument could explain a MOND like frontier even though in a less predictive way as for its exact location. The mean universe density $\bar{\rho}$ should now be dominated by the conjugate one $\bar{\tilde{\rho}}$ by a $1.7^6 \approx 25$ factor if the equality of global densities was reached at the transition redshift $z \approx 0.7$. Yet we know for sure that planets and stars are still gravific meaning that the asymptotic values C^2 and $\frac{1}{C^2}$ of the conjugate metrics did not exchange their roles at the place of such condensed bodies. In other words the existence of static bounded domains anyway implies frontiers delimiting regions in which the cosmological permutation between $a(t)$ and $\tilde{a}(t)$ already occurred and others where it did not. It is not even clear at this stage whether such frontiers are propagating and in the affirmative what determines the location of such frontiers. But anyway such frontier must exist and could be located at the MOND radius in galaxies. Then as we explained in a previous section it should result in the gravitational field from the dark side in the region beyond such radius to be enhanced by a huge factor C^8 relative to the gravity due to our side matter in this region. Eventually this leads to a new picture in which only our side matter can be considered to be significantly gravific below the transition radius while only the dark side matter is significantly gravific beyond this radius. Then because a galaxy on our side implies a slightly depleted region on the dark side by it's anti-gravitational effects, even such a slightly underdense fluctuation on the dark side would result in an anti-anti-gravitational effect on our side. This effect exclusively originating from beyond the transition radius would be difficult to discriminate from the effect of a Dark Matter hallow as an underdense fluctuation in a distribution of negative mass is perfectly equivalent to an overdensity of normal positive mass matter. Also the most spectacular features of Dark Matter and MOND Phenomenology in galaxies such as galaxies that seem to be dominated at more than 99 percent by Dark Matter [21] or unexpectedly high acceleration effects in the flyby of galaxies [24] are more naturally interpreted in a framework where the gravitational effects from the hidden side are dominant beyond the MOND radius.

13. Stability issues about distinct backgrounds: C $\neq$ 1

13.1. *Stability issues in the purely gravitational sector*

Our action for gravity being built out of two Einstein Hilbert terms, each single one is obviously free of Ostrogradsky ghost. This also means that all degrees of freedom have the same sign of their kinetic term in each action.

There might still remain issues in the purely gravitational sector when we add the two actions and express everything in terms of a single dynamical field $g_{\mu\nu}$: everything is all right as we could demonstrate for C=1, but otherwise what we need to insure stability is that in the field equation resulting from the total action, all degrees of freedom will have their kinetic term tilting to the same sign. Again adopting $\bar{h}_{\mu\nu}$ from $g_{\mu\nu} = e^{\bar{h}_{\mu\nu}}$ and $\tilde{g}_{\mu\nu} = e^{-\bar{h}_{\mu\nu}}$ as the dynamical field puts forward that we have exactly the same quadratic (dominant) terms in $t_{\mu\nu}$ and $\tilde{t}_{\mu\nu}$ except that for $C > 1$ (resp $C < 1$) all terms in $t_{\mu\nu}$ are enhanced (resp attenuated) by a C-dependent factor while all terms in $\tilde{t}_{\mu\nu}$ are attenuated (resp enhanced) by a 1/C dependent factor, so that we will find in $t^{\mu\nu} - \tilde{t}^{\mu\nu} \approx C^6 t^{\mu\nu}_{C=1} - C^{-6}\tilde{t}^{\mu\nu}_{C=1} = (C^6 - C^{-6})t^{\mu\nu}_{C=1}$ all such quadratic terms tilting to the same sign, ensuring that the theory is still free of ghost in the purely gravitational sector.

Of course there remains the instability menace in the interactions between matters and gravity which we shall inspect now.

13.2. *Stability issues in the interactions between matter and gravity: the classical case*

Generic instability issues arise again when C is not anymore strictly equal to one. This is because the positive and negative energy gravitational terms $t^{\mu\nu}$ and $\tilde{t}^{\mu\nu}$ do not anymore cancel each other as in the DG C=1 solution. Gravitational waves are emitted either of positive or negative (depending on C being less or greater than 1) energy whereas on the source side of the equation we have both positive and negative energy source terms. Whenever two interacting fields (here the gravitational field and some of the matter and radiation fields) carry energies with opposite sign, instabilities would seem unavoidable (see [26] section IV and V for a basic description of the problem and [27] for a more technical approach) and the problem is even worsen by the massless property of the gravitational field.

Yet, the most obvious kind of instability, the runaway of a couple of matter particles with opposite sign of the energy, is trivially avoided in DG theories [5][8][9][6] [30][31][32][33][34][28] in which such particles propagate on the two different sides of the Janus field and just gravitationally repel each other. Moreover we already have established in section 4.2 the classical stability of the background under small fluctuations at any time. In more details it is straightforward to extend the theory of small gravitational fluctuations to DG in the Newtonian approximation for C=1 (neglecting expansion): the equations governing the decay or grow of compressional fluctuations are :

$$\ddot{\delta\rho} = v_s^2 \Delta\delta\rho + 4\pi G < \rho > (\delta\rho - \delta\tilde{\rho}) \tag{103}$$

$$\ddot{\delta\tilde{\rho}} = \tilde{v}_s^2 \Delta\delta\tilde{\rho} + 4\pi G < \tilde{\rho} > (\delta\tilde{\rho} - \delta\rho) \tag{104}$$

which in case the speeds of sound v_s and $\tilde{v}_s$ would be the same on both sides allows to subtract and add the two equations with appropriate weights resulting in two new equations governing the evolution of modes $\delta\rho^- = \delta\rho - \delta\tilde{\rho}$ and $\delta\rho^+ = \delta\rho + \frac{<\rho>}{<\tilde{\rho}>}\delta\tilde{\rho}$.

$$\Box_s \delta\rho^- = 4\pi G(< \rho > + < \tilde{\rho} >)\delta\rho^- \tag{105}$$

$$\Box_s \delta\rho^+ = 0 \tag{106}$$

Where $\Box_s$ is a fake Dalembertian in which the speed of sound replaces the speed of light. Because $\delta\rho^+$ does not grow we know that $\delta\rho \approx -\frac{<\rho>}{<\tilde{\rho}>}\tilde{\delta\rho}$ and the two can grow according the growing mode of $\delta\rho^-$. The complete study, involving attenuation of gravity between the two sides due to differing scale factors and the effect of expansion will be the subject of the next section. It is already clear that in the linear domain anti-gravity by itself does not lead to a more pathological growth of fluctuations than in standard only attractive gravity: eventually we would expect the growth of a gravitational condensate on one side to proceed along with the corresponding growth of a void in the conjugate side and vice versa[k]. In other words our "instabilities" in the linear domain are nothing but the usual instabilities of gravity which fortunately arise since we need them to account for the growth of matter structures in the universe. These instabilities could be classified as tachyonic (the harmless and necessary ones for the formation of structures), non gradient (fortunately because those instabilities are catastrophic even at the classical level), and ghost (energy unbounded from below which is only catastrophic for a quantum theory) in the terminology of [37] reviewing various kind of NEC violations in scalar tensor theories.

From this it appears that DG is not less viable than GR in the linear domain of small density fluctuations. Again from a field theoretic point of view according [37] the only kind of instability menace that we have are ghost terms which are acceptable for a classical theory : "for ghosts, background is QM unstable but classically stable". This is also confirmed in [91] "we are certain that these perturbations are

[k]The situation is less dramatic than Ref [26] section IV might have led us to think probably because [26] section IV studies the instability of a field that depends on time only (hence it could be a background but we know that our background is not menaced to behave this way since its non ghost interaction always dominates: terms that depends on the larger scale factor dominate and represent a safe interaction that drives the classical behaviour). Also notice that our leading order terms are linear in a gravitational field perturbation h whereas the leading order coupling term is quadratic in the lagrangian (22) of [26] leading to equations of motion of the form $\ddot{\Psi} \propto \Psi^3$.

stable" and in [26] "our phantom model does not predict any significant departures from conventional dark-energy scenarios; in particular, there is no evidence of dramatic instabilities distorting the power spectrum" in which quintessence fields are treated as classical fields.

The real concern will actually arise if in the strong gravity regime, near our pseudo horizon, the local energy density of the dominant source term and that of the gravitational waves can become opposite.

Then for an hypothetical quantized version of the theory such ghost instabilities are of course prohibitive. Then, even at a classical level the real energy exchange between the gravitational field itself (it's kinetic energy quadratic terms) and other fields kinetic energies should start to become significant relative to the Newtonian like energy exchange between kinetic energy of the fields and their gravitational potential energy that drives the evolution of the compressional modes according Eq [103] and [104]. In the strong field regime the problem is thus related to the radiation of gravitational waves when they are carrying non zero energy (for C≠1) while they might couple to matter sources with both positive and negative energies[1].

Anyway, we expect that high density regions produced by compact objects on our side are always in the $C > 1$ domain (remind that the scale factors hence C permutation is triggered at the crossing of densities i.e. wherever the conjugate side density starts to dominate our side density) so that the interaction between this matter and the positive energy gravitational field (due to $C > 1$) is not a ghost interaction at least far enough from the Schwarzschild radius. For the same reason, high density regions produced by compact objects on the dark side are expected to remain in the $C < 1$ domain so that the interaction between the dark side negative energy (from our point of view) matter and the negative energy gravitational field (due to $C < 1$) is again not a ghost interaction.

13.3. *Stability issues in the interactions between matter and gravity: the quantum case*

13.3.1. *Problem statement*

The next step is therefore to try to understand how we might solve stability issues hypothetically arising in the vicinity of our pseudo BH horizon in the quantum case. In the quantized theory the problematic couplings would produce divergent decay rates by opening an infinite space-phase for for instance the radiation of an

[1]This remains true even when great care is being taken to avoid the so-called BD ghost in the massive gravity approach particularly when the perturbations of the two metrics about a common background have different magnitudes i.e. when one parameter of the couple α, β dominates the other in [32]. By the way there is a much worse problem in models having two independent differential equations instead of one to describe the dynamics of two fields assumed independent, i.e. not related from the beginning by a relation such as Eq (1). Then the energy losses through the generation of gravitational waves predicted by each equation are different so that such models are inconsistent [5][8][9][6] [30] as shown in [16].

arbitrary number of negative energy gravitons by normal matter (positive energy) particles. To avoid such instabilities may be the most natural way would be to build the quantum Janus field operator also as a double-faced object, coupling it's positive energy face to usual positive energy particles and it's negative one (from our side point of view) to the negative energy particles (from our side point of view) of the dark side thereby avoiding any kind of instabilities. However the picture described by our classical Janus field equation which in principle (until shown proof to the contrary near the pseudo BH horizon) really allows the direct exchange of energy between GW (with a definite sign of the energy depending on C > 1 or $C < 1$) and matter fields with different signs of the energy does not actually fit into such quantization idea. The most straightforward way to avoid such fatal quantum instabilities if confirmed then would be to consider that the gravity of DG is not a quantum but remains a classical field. Semi-classical gravity indeed treats matter fields as being quantum and the gravitational field as being classical, which is not problematic as far as we just want to describe quantum fields propagating and interacting with each others in the gravity of a curved space-time (within GR) considered as a spectator background. To describe the other way of the bidirectional dialog between matter and gravity i.e how matter fields source gravity, semi-classical gravity promotes the expectation value of the energy momentum tensor of quantum fields as the source of the Einstein equation and this is considered problematic by many theorists. In the last section we shall ask ourselves whether the usual prescription for semi-classical GR i.e. exploiting the quantum fields energy momentum expectation value is the right way to go or if there are more natural alternatives in our case to make quantum and classical fields live together and describe their interactions.

14. Evolution of fluctuations and background (BLK) anisotropies

14.1. *Evolution of fluctuations before* z_{tr}

Given that $\Gamma(t)$ is only non-negligible near t=0, our DG equations are negligibly deviating from GR equations far from t=0 and before the transition redshift. At late times but still before the transition redshift, Dark Matter is required just as in the standard model to have almost the cosmological critical density implied by k=0, the measured value of the Hubble expansion rate and the low density of radiation. Presumably, this Dark Matter did the same good job as within LCDM to help the formation of potentials already in the radiative era and then thanks to these potentials the growth of baryonic fluctuations falling into these potentials. We then have potentially all the successes of CDM phenomenology before the transition redshift with the bonus that we have a new natural candidate for Dark Matter and shall present it in an upcoming section. We also naturally expect almost the same sound horizon at decoupling even though a true singularity is avoided at t=0.

Also remember that the dark side reaches the same density of pressureless matter as on our side at the transition redshift by definition. So the mean dark side density can be extrapolated to extremely small values at high redshift with $\tilde{\rho} \approx z^{-6}\rho = 10^{-18}\rho$ at $z \approx 1000$. For this reason but also more importantly because the dark side matter gravific strength is killed by a huge factor compared to our side matter it is quite obvious that the growth of our side fluctuations starting from $\frac{\delta\rho}{\rho} \approx 10^{-5}$ of the CMB, could not be influenced by the dark side at high z.

As in LCDM, for the evolution of fluctuations the background evolution only becomes important in the matter dominated era arising as usual as an additional friction term $H\dot{\delta\rho}$ where H is the Hubble rate. So we can readily rewrite Eq (103) and (104) taking into account all non negligible effects depending on the scale factor but neglecting sound speeds on both sides assumed to be dominated by non relativistic matter: (see for instance equation (5.1.8) of [42], also written in term of the conformal scale factor for comparison)

$$\ddot{\delta} + H\dot{\delta} = 4\pi G(a^2 < \rho > \delta - \tilde{a}^2 < \tilde{\rho} > \tilde{\delta}) \tag{107}$$

$$\ddot{\tilde{\delta}} + \tilde{H}\dot{\tilde{\delta}} = 4\pi G(\tilde{a}^2 < \tilde{\rho} > \tilde{\delta} - a^2 < \rho > \delta) \tag{108}$$

We here have introduced the relative density fluctuations e.g. $\delta = \frac{\delta\rho}{<\rho>}$. We can justify these equations in the following way. These are as usual sourced by the potential which is just opposite on the dark side relative to our side so δ and $\tilde{\delta}$ are of the same order of magnitudes. Therefore the absolute density fluctuations satisfy $\delta\rho \gtrsim \delta\tilde{\rho}$ before the transition that is as long as $< \rho >$ is greater than $< \tilde{\rho} >$ and the dominance is reversed after the transition. Then inspection of a formula like (56) shows that we can always completely neglect the subdominant terms damped by huge ratios of the scale factors both on the left and the right hand side in order to obtain the potential before or after the transition. As a result, equations (107) and (108) are always valid with an excellent level of approximation.

Those equations confirm that though the dark side gravitational influence on our side can be neglected from the early universe up to the transition redshift (because then $a >> \tilde{a}$), the converse is not true: the dark side is negligibly submitted to it's own gravity but feels the anti-gravitational forces from our side matter structures so:

$$\ddot{\delta} + H\dot{\delta} \approx 4\pi G a^2 < \rho > \delta \tag{109}$$

$$\ddot{\tilde{\delta}} + \tilde{H}\dot{\tilde{\delta}} \approx -4\pi G a^2 < \rho > \delta \tag{110}$$

A common practice is to reformulate those differential equations with derivatives with respect to the scale factor instead of time:

$$\frac{d^2\delta}{da^2} + \frac{3}{2a}\frac{d\delta}{da} \approx \frac{3}{2}\frac{\delta}{a^2} \tag{111}$$

$$\frac{d^2\tilde{\delta}}{d\tilde{a}^2} + \frac{5}{2\tilde{a}}\frac{d\tilde{\delta}}{d\tilde{a}} \approx -\frac{3}{2}\frac{\delta}{\tilde{a}^2} \tag{112}$$

The equation is the usual one for the evolution of our side fluctuations with well known growing solution modes $\delta \propto a$. The second equation is different on its left hand side because in the derivation we use the equations for the background evolution which are different for H ($H^2 = -2\dot{H}$ implying $\frac{\ddot{a}}{\dot{a}^2} + \frac{1}{a} = \frac{3}{2a}$) and $\tilde{H}$ ($\tilde{H}^2 = 2\dot{\tilde{H}}$ implying $\frac{\ddot{\tilde{a}}}{\dot{\tilde{a}}^2} + \frac{1}{\tilde{a}} = \frac{5}{2\tilde{a}}$). This second Meszaros kind of equation can also be obtained with derivatives with respect to the scale factor a instead of $\tilde{a}$.

$$\frac{d^2\tilde{\delta}}{da^2} - \frac{1}{2a}\frac{d\tilde{\delta}}{da} \approx -\frac{3}{2}\frac{\delta}{a^2} \tag{113}$$

Here there is not only the particular solution driven by the source $\tilde{\delta} = 3\delta \propto a$ (notice that the conjugate fluctuations are not opposite in this linear regime) but also an instability i.e. an additional growing solution of the homogeneous (without source) equation: $\tilde{\delta} =\propto a^{3/2}$. In the radiative era for our side (cold for the dark side) the equation was $\frac{d^2\tilde{\delta}}{da^2} - \frac{1}{a}\frac{d\tilde{\delta}}{da} =\approx -\frac{3}{2}\frac{\delta}{a}$ and the self growing instable mode was $\propto a^2$ while the leading driven mode was $\propto a \ln a$. The dominant instable mode growing as a^2 is also active beyond the horizon. So either the initial conditions at t=0 was such that the power on the dark side was extremely small compared to the power on our side on all scales of interest and extremely fine tuned to remain so in the subsequent evolution which is probably an unrealistic hypothesis, or it was comparable and then all scales of interest have left the linear regime very soon in the primordial universe. We will nevertheless run two kind of simulations starting at z=63, a simulation assuming fine tuned initial conditions so we can neglect the instable self growing mode and confront the Nbody simulation results to the linear regime predictions of our differential equations: this is only to validate our Nbody evolution equations. Then a second kind of simulation with various powers of the dark side relative to our side and various shapes (flat or very red or very blue) of the initial power spectrum at z=63 or a higher z=630: then instead of generating directly a high power at z=63 that would violate the linearity conditions for Ngenic we can generate a power still in the linear regime but at a higher z=630 and then study how the simulation makes it evolve up to the non linear regime.

We anticipate that our main result from the Nbody simulation will be that anyway, entering the non linear regime will produce a fast decay of those fluctuations power relative to the power on on our side on a large range of scales, may be up to the largest scales we are able to probe. Eventually, the power reached on the dark side is frozen in a specific k^3 law (always the same) and therefore no longer sensitive to initial conditions for $\tilde{\delta}$. This not only confirms that after leaving the linear regime the dark side fluctuations are not able to grow further under their negligible self-gravity but also tells us that all gravity induced fluctuations (large scale bulk flows) in the linear regime are eventually destroyed when entering the non linear regime i.e. turned into a white noise, thermalized, except may be at very small scales and only N body simulation could help us understand in detail this evolution.

May be it's only on the shortest scales that fluctuations opposite to our side fluctuations may have grown non linearly and remained and those would be the new seeds for the growth of fluctuations on the dark side in the future. If there were such short scales voids everywhere we have our side over-densities, of course $\tilde{\delta}^- \geq -1$ and the growth factor of those voids should asymptotically tend to 0 so many small scale dark side voids would already have reached a density contrast close to -1 at which point they have no more ability to significantly grow anymore.

To study this we have adapted the N-body simulation code Gadget-4[[118]][[119]], still assuming LCDM in a first step for the evolution of our side background and the corresponding $\tilde{a} = 1/a$ evolution. We also don't yet simulate any transition that would allow the dark side fluctuations to become the new driver of fluctuations growth, as this study is postponed to the next subsection. We started our pure DM (no baryons) simulation at z=63 in a 500 Mpc box and a total of 256^3 particles of both types alternating in the grid with the usual initial conditions on our side as determined by Planck and amplitude of fluctuations 3 times greater on the dark side in the first simulation, as if their growth had been driven mainly by our side rather than by their own instable mode in the primordial universe. In a second simulation we start the simulation with an amplitude of fluctuations 60 times greater on the dark side (but at a ten times higher redshift of 630 to be sure to start the simulation in the linear regime).

To simulate the dark side we have simply assigned zero mass to it's particles giving them the status of mere test particles in the gravitational potentials produced by our side. We verify that the power spectra of our side matter is then unaffected as expected.

The equations for the time integration of the dark side collisionless particles need to be modified accounting for their contracting scale factor. In comoving coordinates the conjugate momenta p (our side) and $\tilde{p}$ (dark side) satisfy $p = av_{pec} = a^2 \frac{dx}{dt'} = a\frac{dx}{dt}$, $\tilde{p} = \tilde{a}\tilde{v}_{pec} = \tilde{a}^2 \frac{d\tilde{x}}{dt'} = \tilde{a}\frac{d\tilde{x}}{dt}$ with t the conformal time coordinate, t' and $\tilde{t}'$ the standard time coordinates while x and $\tilde{x}$ are spatial comoving coordinates of bodies respectively on our and the dark side. [m] The peculiar velocities which are also the comoving conformal time velocities are actually the velocities (and corresponding displacements) generated by NGENIC for the initial conditions and a 3 times greater amplitude of $\tilde{\delta}$ (power multiplied by 9) on the dark side with respect to our side simply means that such velocities and displacements are also multiplied by 3 on the dark side.

The drift equation satisfied by p and applied at each simulation da step is :

$$d\mathbf{r} = \mathbf{p}\frac{da}{a^3 H(a)} = \mathbf{p}\frac{da}{a^2 \mathscr{H}(a)} \tag{114}$$

[m]the different notation and the word comoving should not confuse us : the comoving coordinate system is a common spatial coordinate system for both sides, i.e. both sides galaxies have fixed spatial coordinates in this system even though one side is contracting and the other is expanding! : this is explicitly the meaning of our two conjugate metrics that we wrote from the beginning in one and the same comoving coordinate system.

and therefore on the dark side:

$$d\tilde{\mathbf{r}} = \tilde{\mathbf{p}} \frac{d\tilde{a}}{\tilde{a}^3 \tilde{H}(\tilde{a})} = \tilde{\mathbf{p}} \frac{d\tilde{a}}{\tilde{a}^2 \tilde{\mathscr{H}}(\tilde{a})} = \tilde{\mathbf{p}} \frac{da}{aH(a)} \tag{115}$$

The kick equation is

$$d\mathbf{p} = -\nabla\Phi \frac{da}{a^2 H(a)} \tag{116}$$

where Φ is not the scalar potential entering in the perturbed metric in Newtonian Gauge but rather a redefined "comoving" Potential which is just the scalar potential times a. But the gradient of this comoving potential is the one that is actually computed at each step in the simulation from the distribution of gravific masses. It remains that the corresponding gradient on the dark side, due to this redefinition is not simply the opposite but rather $-\frac{\nabla\Phi}{a^2}$, from which follows the dark side kick equation

$$d\tilde{\mathbf{p}} = \frac{\nabla\Phi}{a^2} \frac{d\tilde{a}}{\tilde{a}^2 \tilde{H}(\tilde{a})} = \nabla\Phi \frac{da}{a^4 H(a)} \tag{117}$$

The dimensionless matter power spectra we obtain at various redshifts for our and the dark sector and their ratio are shown in Fig. 14. The large power fluctuations at high redshift and small scales are due to the shot noise and peaks at multiples of our grid Nyquist frequencies. We find that in the linear domain the dark side growth of fluctuations remains very close to our side growth i.e. evolves like the expected $\tilde{\delta} \approx 3\delta \propto a$ at startup. But then is progressively slowing down and after several e-folds of the scale factor accelerates again on the largest scales.

To better understand what's going on we run a simulation in an extremely linear regime i.e. with a super low σ_8 =0.0005 as input to be sure that no nonlinear effect is influencing the growth of fluctuations (see Fig 15) and notice that in the late acceleration phase we get close to the $\tilde{\delta} \propto a^{3/2}$ law even in a pure CDM simulation (no Lambda). How to interpret this ?

The natural interpretation of this behavior is that our approximate initial conditions at z equal 63 imply that we are actually in a transient regime: fluctuations initially don't grow exactly as $a(t)$ even though we tuned our initial conditions for that and because of this a $a^{3/2}(t)$ component is excited apparently with an opposite phase so that the growth is momentarily slowing down but soon after it will start to dominate and produce a faster growth converging to a $a^{3/2}(t)$ law. This transition is even clearer in simulations with initially slightly different initial conditions: a boosted growth by increasing initial velocities by 10 percent.

Having a confirmation that our N-body simulations can reproduce results expected from differential equations in the very linear regime gives us confidence in applying our kick and drift rules in our two simulations with realistic σ_8 and follow them in the non linear regime first with our simulation assuming initial dark side

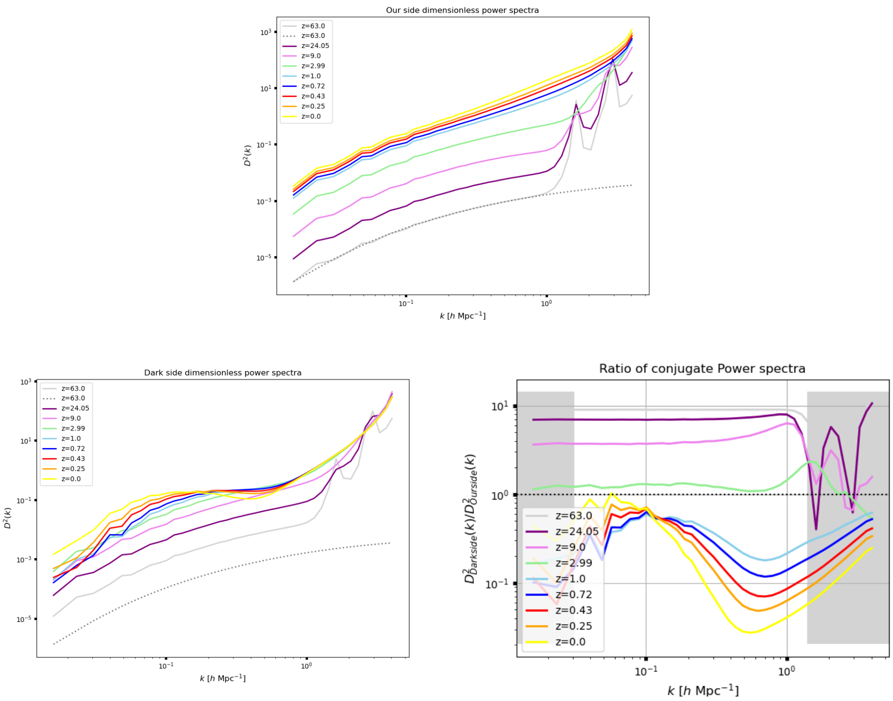

Fig. 14. Conjugate power spectra for $\tilde{\delta} \approx 3\delta$ initial fluctuations at z=63

fluctuations at z=63 satisfying $\tilde{\delta} \approx 3\delta$, then the second simulation assuming initial dark side fluctuations at z=630 satisfying $\tilde{\delta} \approx 60\delta$.

In the first simulation (Fig. 14) we check that as far as we remain in the linear regime (from z=63 to $z \approx 9$) the ratios are flat and growing uniformly for all scales relevant for the σ_8 parameter in between the two gray bands. But then non linear effects start to spoil this behavior at smaller redshifts. The ratio of powers first starts collapsing on small scales probably because dark side fluctuations can't grow by themselves and because their driver are our side fluctuations which they can't follow in the very non linear regime. This effect then seems to propagate to larger and larger scales ($k < 0.08 Mpc^{-1}/h$) even those which are still in the linear domain. We notice that those power spectra near the transition redshift are already in the (between 0.1 and 1) non linear regime for the σ_8 scales.

In the second simulation we observe in Fig. 16 the same effect occurring sooner and freezing the dark side power spectrum in a pure white noise spectrum: $P(k) \propto k^3$ for the dimensionless power spectrum up to the largest scales: this can be interpreted as the effect of destruction of the fluctuations inherited from coherent bulk flows at all but may be the smallest scales where the power seems to increase faster than k^3 as on our side. We started the simulation here at z=630 to insure linear conditions

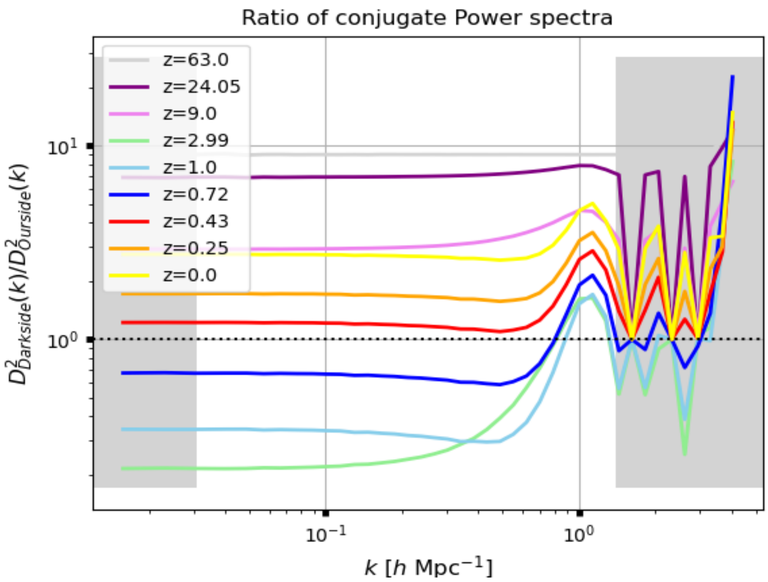

Fig. 15. Ratio of conjugate power spectra for a super low σ_8 =0.0005 and $\tilde{\delta} \approx 3\delta$ initial fluctuations at z=63

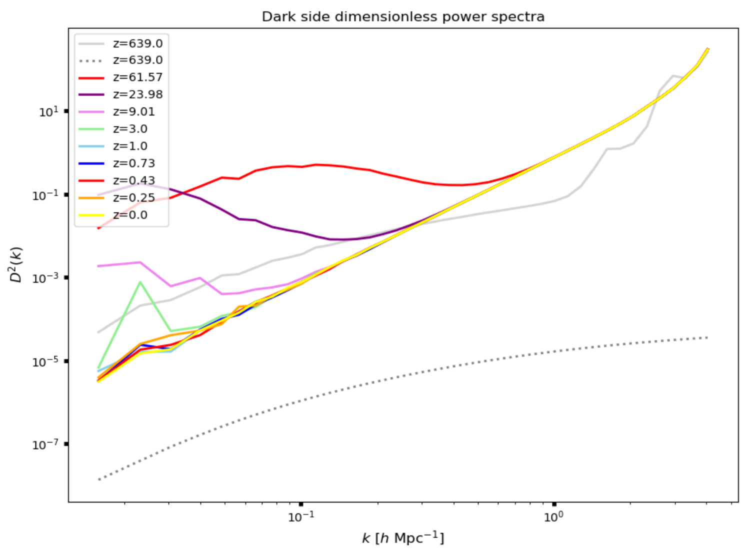

Fig. 16. Dark side power spectra for $\tilde{\delta} \approx 60\delta$ initial fluctuations at z=630

even though we started with 60 times higher amplitude fluctuations. The crucial behavior, i.e. a thermalization of the power spectrum was confirmed in many other simulations with very different initial shapes and powers.

This fate of the power spectrum is realistic if the dark side initially started to contract with a power spectrum non completely negligible and fine tuned relative to our side power spectrum at the contrary to the first simulation in which some power remains on the largest scales. We will say more on the white noise spectrum in a forthcoming subsection.

Of course a transition is going to take place somewhere around z=0.7. So the spectrum plotted at z=0.72 gives us a good approximation of what is going to be the new dark side initial power spectrum (apparently a pure white noise at all except smallest scales) at transition redshift that will drive the subsequent evolution and power spectra plotted at lower redshift are irrelevant so far.

14.2. *Evolution of fluctuations after* z_{tr}

It remains to investigate the influence of fluctuations from the dark side after the transition redshift. For that we need to rely on the extremely efficient effect of the scale factors permutation to understand the gravitational effect of dark side fluctuations (voids) starting to play a significant role and produce the MOND empirical laws in galaxies. But in accordance with what we also explained earlier we have two kinds of regions for fluctuations : those static regions around our side concentrations of baryonic matter in which the gravity from our side $\delta\rho_{static}$ remains hugely enhanced over the gravity from the dark side $\delta\tilde{\rho}_{static}$ because the scale factor was not re-normalized there, and the rest of the universe in which at the contrary, it is the gravity from the dark side $\delta\tilde{\rho}_{evol}$ that hugely dominates that from $\delta\rho_{evol}$. Close to the transition redshift, we would therefore expect similar strengths for $\delta\rho_{static}$ and $\delta\tilde{\rho}_{evol}$ gravity, however since the static domains are likely to house highly non linear and very short scales fluctuations we decide, at least in a first step, not to include them in the simulation. The static domains content represents a small percent of the total density of the universe, likely less than 1 percent, so neglecting them in the analysis should hopefully not significantly change the evolution of the much larger scales of our power spectra. Remember however that those very small scale fluctuations house the tracers for all cosmological analysis (galaxies for clustering and RSD studies, halos=lenses giving their contribution to lensing analysis) and any effect reducing the ability of the tracer to be detected can reduce the large scale measured powers by the same global factor, just as an additional bias parameter. This is probably the case for the lenses: the lensing power of the peripheral region of each individual lens (region lying outside the static domain, thus in the cosmological domain) will be canceled at the transition redshift, probably dramatically reducing it's total lensing power in DG unless the dark side takes over in producing the required lensing power, but this is less likely the case for the luminosity of visible tracers.

Since we are already in the mildly non linear regime and in a transient regime on the dark side the analysis of our linear differential linear equations after transition redshift would not be very instructive. Here are our expectations before starting the simulations: the dark side fluctuations are now submitted essentially to their own gravity and in a contracting background are naively expected to grow faster. A somewhat higher growth factor of the dark side fluctuations in the late universe is interesting as it might help produce anomalies of our side voids growth factor exceeding LCDM expectations specially at low redshift as has indeed been reported for instance for cosmic voids below z=0.4[63]. Moreover the dark side matter is expected to cluster non linearly near the center of our voids producing an increasing repelling force on our side nearby matter replacing the own repulsive effect of our side voids that has been switched off following the transition redshift. This take over is necessary for the lensing power of dark side fluctuations to take over for instance in cosmological voids at redshifts smaller than the transition redshift.

Remember however that $\delta^- \geq -1$ so the growth factor of our voids should start to decrease in the future and asymptotically tend to zero. On the other hand, the dark side under-densities on short scales must also have relayed as gravific actors our side, now switched off, over-densities for the same reasons. And then the resulting growth should remain much smaller corresponding to dark side voids that cannot grow anymore having almost reached the limit -1 for $\tilde{\delta}^-$.

These considerations should also motivate a serious re-investigation within our framework of numerous recently reported anomalies of the growth rates [64] [65] and is also a plausible origin for the mild tensions between some of our predicted and observed BAO points at low redshift due to people influenced by GR and the standard model expectations not correctly understanding the shape of BAO peaks even in the linear regime. On the smallest scales, fluctuations $\tilde{\delta}$ in the dark side distribution are also expected to produce gravific effects mimicking so well the gravity of DM halos that those are probably wrongly attributed to Dark Matter Halos within LCDM. Indeed, around our galaxies, the voids that formed before the transition redshift on the dark side have started to exert their confining force helping the rotation of galaxies after the transition redshift, as these exactly behave as dark matter halos but without cusps, from our side point of view. Again such effect must replace that of genuine dark matter that was gravitationally active before but not anymore after the transition redshift.

We are now going to see that our N-body simulation results oblige us to dramatically reconsider all these expectations if we want to stick to the hypothesis of Cold Dark matter being made of heavy particles or bodies on the dark side : the basis of our N-body simulation. We indeed pursued our N-body simulations in the accelerated universe to see what's actually going on following the transition redshift. Now the drift equations remain the same but our side particles become the test particles in the gravity produced by the dark side particles so the kick equations get modified:

$$d\tilde{\mathbf{p}} = -\nabla\tilde{\Phi}\frac{d\tilde{a}}{\tilde{a}^2\tilde{H}(\tilde{a})} = -\nabla\tilde{\Phi}\frac{da}{a^2H(a)} \tag{118}$$

where $\tilde{\Phi}$ sourced by the darkside gravific masses, which gradient is computed by Gadget-4, is now the dark side GR scalar potential times $\tilde{a}$. The corresponding gradient on our side, is now $-\nabla\tilde{\Phi}a^2$, from which follows our side kick equation

$$d\mathbf{p} = \nabla\tilde{\Phi}a^2\frac{da}{a^2H(a)} = \nabla\tilde{\Phi}\frac{da}{H(a)} \tag{119}$$

We show the ratios between our side power spectra and the LCDM predicted one now for three possible new initial conditions corresponding respectively to transition redshifts 0.67, 0.72, 0.75 in Fig. 17.

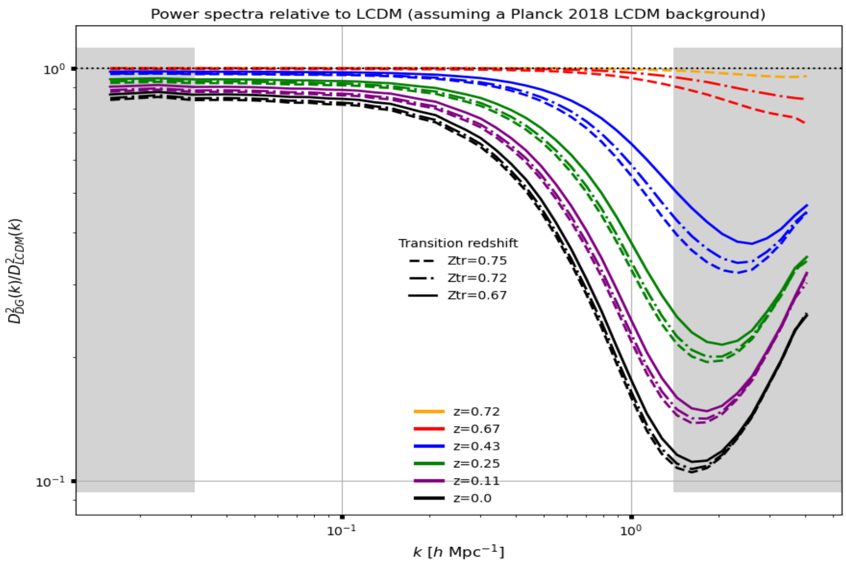

Fig. 17. Ratio of LCDM and DG predicted power spectra

We here chose our second previous simulation to give us the new initial conditions but could check that the final results are almost insensitive to this choice in Fig. 18.

Again the important scales are those between the gray bands that are actually probed and integrated over to get the σ_8 parameter. And the probably most important power spectrum is the z=0.43 one as it is in the middle of the range of scales that are the most accurately probed by current weak lensing and clustering analysis. Also remember that we assumed LCDM for the background so we are only studying here the effect of the new fluctuations dynamical laws induced by the transition. Yet we know that a significant deviation from LCDM must also come from a different background evolution in particular before the transition redshift when there

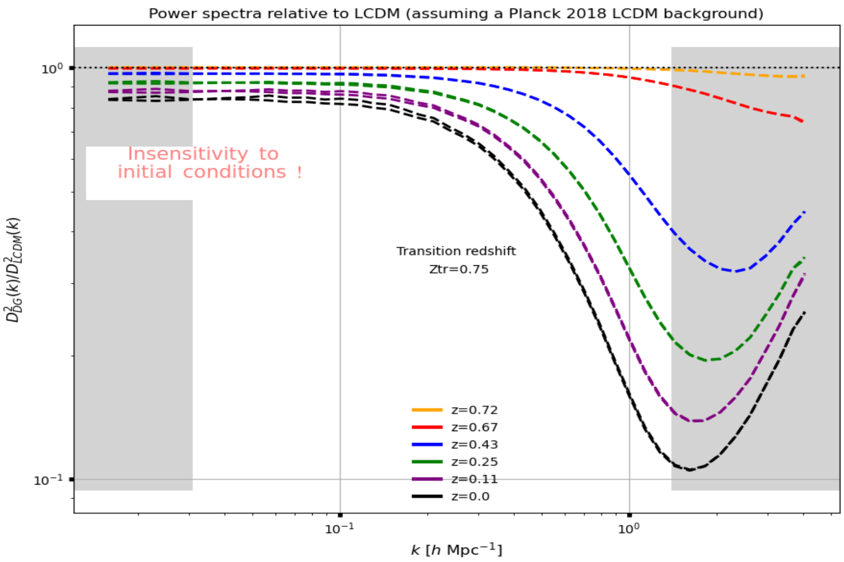

Fig. 18. The same to show the small effect of a big arbitrary change in the Initial Conditions: two black curves (corresponding to two different ICs) at z=0 only start to deviate from each other very slightly on large scales

is actually no dark energy in DG: this must have enhanced the fluctuations at high z, an effect which is completely neglected here. Instead there is no departure from LCDM before the transition redshift in this partial analysis. So the deficit in power that we see now on intermediate scales in power spectra at z=0.43 could partly be compensated by a higher power inherited from the era before the transition. Notice that the ratios are unsurprisingly closer to 1 for a later transition but in all cases we have a very small lack of power already on large scales exceeding $0.08Mpc^{-1}/h$ but becoming more serious on smaller scales. For $\sigma_8(z=0)$ this produces a roughly 10 percent deficit for all plausible transition redshifts. Again the effect is slightly lower for a later transition redshift.

It should affect all probes identically: clustering, weak lensing and RSDs, and is interesting to explain the low σ_8 anomaly: a deficit should mainly be seen at redshifts lower than the transition redshift, the smallest scales entering in σ_8, while at the contrary an excess of power should be detected beyond z_{tr} because of the different DG background evolution. Notice that the deficit of power increases importantly at smaller less well probed redshifts.

Again the importance of these last results is that they are very robust to any, even very important change in the initial conditions on the dark side which we don't know. In this scenario our previous expectations are contradicted in the sense that since the transition, the dark side did not have enough time, except may be at the smallest scales, to reconstitute significant new fluctuations that could take over. So, eventually even though we have interesting results for some of the observable

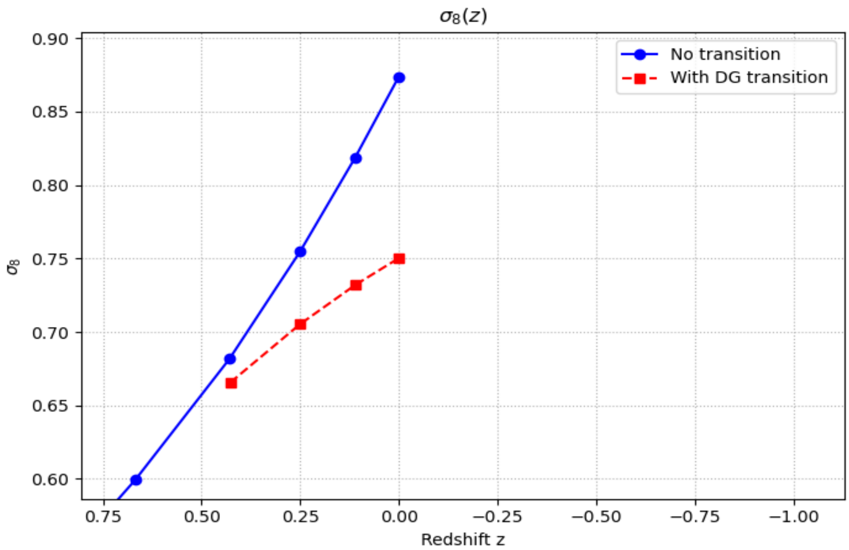

Fig. 19. DG vs LCDM σ_8 evolution

such as σ_8 obtained from clustering, that might give some credit to DG, it remains that anyway, as shown in the previous subsection, except for extremely fine tuned ICs, weak lensing is expected to be almost completely suppressed due to the much smaller power spectrum on the dark side at the end of the simulation and is not able to take over which is clearly a dead end for this Dark side Cold Dark Matter scenario.

14.3. *Scalar field DM as an alternative scenario*

The reason why our conventional DM hypothesis failed, as shown by Nbody simulations, is first that the darkside fluctuations grow more than on our side, acquire greater velocities in the linear regime but then, in the non linear regime in the absence of self gravity are converted into thermal velocities of particles, much too high to allow those particles to be trapped by our side gravitational potentials. These bodies are essentially free streaming after the flows become incoherent i.e. following shell crossing that triggers multi-streaming and then violent relaxation (fast variations of gravitational potentials allowing fast energy exchange between particles and thermalization). A scalar field has a different phenomenological behavior and some of its properties naturally address those issues.

- Scalar field = coherent wave, not particles: A scalar field obeys the Klein–Gordon or Schrödinger–Poisson equations and possesses a coherent phase, dispersion, and wave interference. CDM particles possess none of these properties.

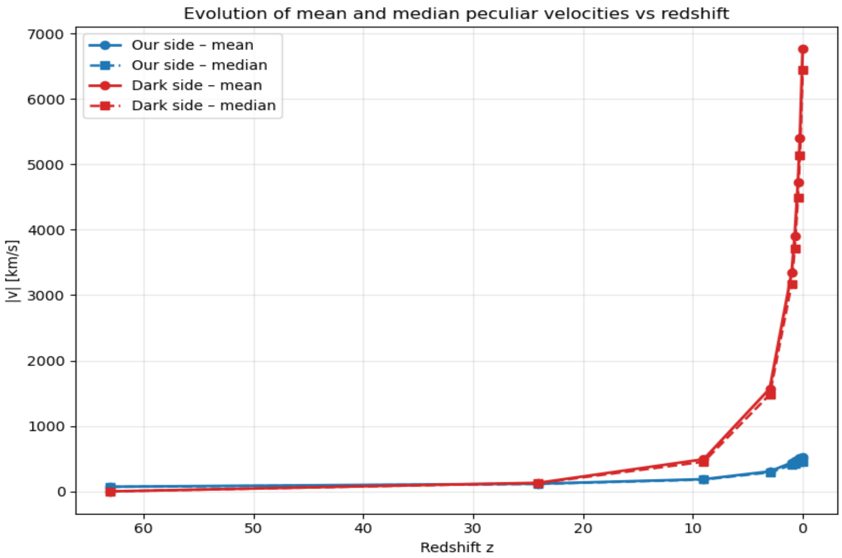

Fig. 20. mean velocities on our and the dark side

- Unique velocity field vs. multistreaming: In the Madelung representation

$$\psi = \sqrt{\rho}\, e^{iS/\hbar}, \qquad \mathbf{v} = \frac{\hbar}{m}\nabla S,$$

the scalar field defines a single-valued, continuous velocity field. CDM inevitably produces multistreaming and shell-crossing which are incompatible with scalar-field dynamics.

- Quantum pressure is unavoidable: The Madelung equations contain the quantum pressure term

$$Q = -\frac{\hbar^2}{2m^2}\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}},$$

which suppresses small-scale structure and prevents cusp formation. CDM lacks this term entirely and can be reproduced by SFDM only at larger scales.

- No thermalisation: in SFDM Schrödinger evolution is linear and conserves phase-space structure; there is no violent relaxation. CDM N-body dynamics always thermalises after shell-crossing, producing behavior absent in scalar-field models. Our case is worse because in the absence of self gravity, thermalisation propagates to all scales.

SFDM is therefore now the best candidate to allow fluctuations on the dark side to settle in the state $\tilde{\delta} \approx -\delta$ which is mandatory to get the correct lensing observables just after transition at least for large scale voids. Not only this but if SFDM is the only viable DM for the dark side, it might also be a clue that SF explains DM also on our side but not necessarily with the same mass.

14.4. *The Schrodinger equation for the dark side SFDM*

We introduce a massive scalar field on the dark side with the quadratic potential $V(\tilde{\phi}) = \frac{1}{2}m^2\tilde{\phi}^2$ and $m >> H$ as this is the standard choice to get the equation of state w=0 for the background, and the cosmological behavior of collisionless dark matter. In the non relativistic regime (weak gradients and speed of the field), we can decouple a fast oscillation term $e^{-im\tilde{t}}$ from a slowly varying envelope $\tilde{\Psi}$ of the field satisfying the Schrodinger equation here given in physical coordinates ($\tilde{r}$,$\tilde{t}$) in presence of a gravitational field with potential $\tilde{\Phi}$:

$$i\hbar\frac{\partial}{\partial\tilde{t}}\tilde{\Psi} = [-\frac{\hbar^2\nabla^2_{\tilde{r}}}{2m} + m\tilde{\Phi}]\tilde{\Psi} \tag{120}$$

This setup can either represent a classical scalar field or a fuzzy Dark Matter made of particles in which case the quadratic potential is natural.

According the Madelung decomposition $\tilde{\Psi} = \sqrt{\tilde{\rho}}e^{i\tilde{S}}$ the squared modulus of $\tilde{\Psi}$ represents the density while the spatial gradient of it's phase $\tilde{S}$ gives us the speed of the associated fluid $\tilde{v} = \frac{1}{m}\nabla_{\tilde{r}}\tilde{S}$. Indeed inserting it in the Schrodinger equation, the real and imaginary part equations are nothing but the familiar continuity and Euler equations except for an additional quantum pressure term in the latter:

$$\partial_t\tilde{\mathbf{v}} + (\tilde{\mathbf{v}}\cdot\nabla_{\tilde{r}})\tilde{\tilde{\mathbf{v}}} = -\nabla_{\tilde{r}}\tilde{\Phi} + \underbrace{\frac{1}{2m^2}\nabla_{\tilde{r}}\left(\frac{\nabla^2_{\tilde{r}}\sqrt{\tilde{\rho}}}{\sqrt{\tilde{\rho}}}\right)}_{\text{quantum pressure force}}$$

preventing the growth of small scale structures. Notice that this dark side dark matter should only start to produce observational effects when it becomes gravitationally active i.e. after the transition redshift near 0.7 to the accelerating universe and therefore evades the by far most constraining existing limits on m (Ly-α forest measurements between z=2 and 5). Before the transition redshift the field is understood to be gravitationally passive, i.e. a test scalar field in the inverted gravitational field (opposite gravitational potentials and inverted scale factor) created by our side content and we now need the Schrodinger equation in the common comoving spatial coordinate system $\mathbf{x}$ for both metrics such that $\mathbf{r} = a\mathbf{x}$ $\tilde{\mathbf{r}} = \tilde{a}\mathbf{x}$. If the scalar field was in our side metric the Schrodinger equation would read:

$$i\hbar\frac{\partial}{\partial t}\Psi = [-\frac{\hbar^2\nabla^2_x}{2ma^2} + m\Phi]\Psi \tag{121}$$

but the SFDM here lives on the dark side which clocks read a different standard time $\tilde{t}$

$$i\hbar\frac{\partial}{\partial\tilde{t}}\tilde{\Psi} = [-\frac{\hbar^2\nabla^2_x}{2m\tilde{a}^2} + m\tilde{\Phi}]\tilde{\Psi} \tag{122}$$

In conformal time $d\eta = \frac{dt}{a} = \frac{d\tilde{t}}{\tilde{a}}$ and using $\tilde{a} = \frac{1}{a}$ the equations read

$$i\hbar\frac{\partial}{\partial\eta}\Psi = [-\frac{\hbar^2\nabla_x^2}{2ma} + ma\Phi]\Psi \tag{123}$$

$$i\hbar\frac{\partial}{\partial\eta}\tilde{\Psi} = [-\frac{\hbar^2 a\nabla_x^2}{2m} - \frac{m}{a}\Phi]\tilde{\Psi} \tag{124}$$

as we know that $\tilde{\Phi} = -\Phi$ created by our side matter. It remains to express the later equation in our side standard time because this is where we observers and our clocks are living.

$$i\hbar\frac{\partial}{\partial t}\tilde{\Psi} = [-\frac{\hbar^2\nabla_x^2}{2m} - \frac{m}{a^2}\Phi]\tilde{\Psi} \tag{125}$$

to be compared with equation for SFDM on our side 121 in same coordinates.

We will also need the equations for the evolution of fluctuations derived from the schrodinger equations, here remaining in conformal times for practical reasons.

$$\frac{d^2\delta}{d\tau^2} + \mathscr{H}\frac{d\delta}{d\tau} + \left(\frac{\hbar^2k^4}{4m^2a^2} - 4\pi Ga^2\bar{\rho}\right)\delta = 0 \tag{126}$$

$$\frac{d^2\tilde{\delta}}{d\tau^2} - \underbrace{\mathscr{H}\frac{d\tilde{\delta}}{d\tau}}_{1} + \underbrace{\frac{\hbar^2k^4a^2}{4m^2}\tilde{\delta}}_{2} + \underbrace{4\pi Ga^2\bar{\rho}\,\delta}_{3} = 0 \tag{127}$$

This is because we will want to estimate the domains of prevalence of the three terms: the first that drives the dominating solution of the homogeneous equation: $\tilde{\delta} \propto a^{3/2}$ as for CDM, the second is the new proper SFDM pressure term dominating on short scales, the third is the gravitational term which, as for CDM, naturally becomes subdominant in the linear regime relative to the first one as it produces a smaller growth factor of the inhomogeneous equation solution, but should emerge and eventually dominate in the non linear regime. Equating first and second term we can easily derive a k-frontier we name $k_{hp} = \sqrt{\frac{3mH}{\hbar}}$ between hubble and pressure term that will be shown as a point in various spectra and correlation curves. Equating third and second term we can derive another much less trivial k-frontier k_{pg} between pressure and gravity: the corresponding point will only be shown at the end redshift of the simulation, usually 0.7, at which for any k, the needed δ_k in the source term is known from the standard Gadget4 LCDM simulation and $\tilde{\delta}_k$ from the SFDM simulation itself. The two points (k_{hp} and k_{pg}) turn out to be very close to each other which can be justified analytically and simply when $\delta_k \approx \tilde{\delta}_k$ in the concerned k range.

14.5. *SFDM Simulation*

With the aid of ChatGPT we have simulated the evolution driven by the Schrodinger equation in 2d with a python code. The potentials are provided by the previous

Gadget4 CDM simulation and assumed fixed all along the simulation from z=20 to z=0.7, the assumed transition redshift: indeed the potentials do not evolve in a matter dominated universe in the linear regime and this could be checked with Gadget4. As our simulation stops at z=0.7, for the scales we are interested in, the linear regime is a very good approximation because our goal is not to give an accurate prediction (not possible as the initial conditions are not known on the dark side) but to show that the simulation eventually naturally terminates with the correct order of magnitude for the dark side fluctuations for them to be able to take over after the transition redshift.

Special care and other python code is required for the generation of appropriate Initial Conditions at z=20. In a contracting universe in the linear domain, for scales for which we can neglect quantum pressure we are back to the same evolution equation of the density contrast that we had in the CDM scenario. It has, both below and beyond the horizon, a dominant growing mode (instability) solution of the homogeneous equation $\tilde{\delta} \approx a^{3/2}$. The possibility of a growing mode beyond the horizon is often ignored in cosmologies closer to the standard paradigm because in expanding universes, the homogeneous equation solutions decay. But in our case, it grows and is of major importance.

If we assume initial primordial fluctuations of the SFDM field correlated with the primordial potentials at the Big Bang (a bidirectional gravitational interaction between the two sides, i.e. both sectors gravitationally active in a primordial static state of our conjugate universe $a = \frac{1}{\tilde{a}} = 1$ could have produced this, see our section devoted to our alternative idea to inflation). Once those fluctuations suddenly left the horizon as a result of the conjugate universes starting to expand/contract thanks to the Γ mechanism, only the $\tilde{\delta} \approx a^{3/2}$ could start to drive the evolution of those fluctuations and then after reentering the horizon, still on scales for which pressure can be neglected, this mode should have remained the dominant one until reaching the non linear domain. This is fortunate because the mode driven by the source would have reversed the correlation as we have seen earlier for CDM ($\tilde{\delta} \approx 3\delta$ in this case i.e. dark side fluctuations are not opposite to our side fluctuations). We therefore want to simulate ICs coherent with the dominant growing mode at z=20 $\tilde{\delta} \approx a^{3/2}$ that should hopefully preserve the initial correlations (dark side over density for a dark side potential well). Even though the initial power spectrum we simulate is arbitrary in power and shape at this stage, we know that the speed of the fluid, the phase of the field and the density contrast are related. On our side we would have, from the continuity equation in comoving coordinates and in Fourier space ($\nabla^2 \rightarrow -k^2$):

$$\frac{\partial \delta_k}{\partial t} - k^2 \frac{\hbar}{ma^2} S_k = 0 \tag{128}$$

but on the dark side the a^2 disappears:

$$\frac{\partial \tilde{\delta}_k}{\partial t} - k^2 \frac{\hbar}{m} \tilde{S}_k = 0 \tag{129}$$

and the dominant growing mode imposes $\dot{\tilde{\delta}}_k = \frac{3}{2}H(a)\tilde{\delta}_k$ so that eventually

$$\tilde{S}_k = \frac{3}{2}H(a)\frac{m}{\hbar k^2}\tilde{\delta}_k \tag{130}$$

is what we need to generate coherent ICs (phase and modulus of $\tilde{\Psi}$).

What we would like to confirm from the simulation is that the dominant instable growing mode will not destroy the initial phase correlation between potentials and initial $\tilde{\delta}$ both during the linear evolution and after living the linear regime at least for all those scales that are not prevented to grow or affected in any way by the quantum pressure. Notice that on their own side the potentials are understood to be still strongly phase correlated with their initial states: potentials not only are frozen in our cold era but are still sourced by CDM fluctuations as those that sourced the potentials in the primordial state, and since the present CDM fluctuations have not loose their phase correlation with respect to the initial ones, so have the potentials: even though the correlation has been lost momentarily during the radiation era, it has been recovered in the matter dominated era.

The second thing we would like to confirm is that the fluctuations of the SFDM will freeze near $\tilde{\delta} = 1$, as they are passive and cannot grow further non linearly under their own gravity (no self gravity on the dark sector before the transition redshift): the SFDM of the dark side can only settle in the voids of our side distribution which themselves have δ of the order of one since the transition redshift (this is also the order of magnitude of the σ_8 parameter). In other words, we have reasons to hope that at the transition resdhift fluctuations from the dark side could seamlessly gravitationally take over our side fluctuations on a large range of voids scales and explain void lensing measurements.

14.6. *Simulation results*

Our codes have been thoroughly tested in the standard SFDM scenario with the scalar field on our side to insure that we are able to recover standard results in this case for the growth rate as in the CDM scenario.

The SFDM simulation with scalar field on the dark side requires the inverted potentials from the Gadget4 CDM only (no baryons) simulation as input on reliable not too non linear scales i.e. on cosmological scales with $k_{max} \approx 40h/Mpc$. This defines the order of magnitude of the Nyquist frequency of the SFDM simulation grid we need i.e. 2πN/L. And then because the number of nodes N^3 in the grid is limited by the amount of maximum available RAM (128 Go for N=8192 on a PC), $2\pi/L \approx k_{min} \approx 4.10^{-3}h/Mpc$. The SFDM mass having it's transition between pressure and gravity domination well within the range of scales $k_{min} \leq k \leq k_{max}$ between z=20 and z=0.7 and that we can simulate in a reasonable CPU time is of the order of $10^{-27}eV/c^2$. For these practical reasons, we have simulated this but also lower masses ($10^{-28}eV/c^2, 10^{-29}eV/c^2$) which was also possible by extrapolating the input power spectrum of potentials to larger scales (smaller k_{min}) relying on the

LCDM robust prediction on such very linear scales. Remember however that we have actually no physically preferred mass for our scalar field. So the true SFDM mass might be orders of magnitude larger than the ones chosen here for practical reasons only. We also had to restrict to 2d only for a reasonable simulation time.

The ICs SFDM adimentional power spectrum shape is arbitrarily chosen flat in 3d (red in 2d). But the power of the ICs at z=20 is limited by the non wrapping condition that phase differences from cell to cell (this can always be reduced by increasing N) evaluated from Eq 130 should never exceed π otherwise we would have phase wrapping. This is a serious limitation because if we start at a too low power the simulation will hardly reach the nonlinear regime we want to study before the end redshift i.e. the z=0.7 transition redshift. Specifically it is the initial power at $k_{min} = \frac{2\pi}{L}$ i.e. the largest scales which are the less affected by pressure that we want to increase as much as the no-wrapping requirement makes it possible and from Eq 130 we see that such scales are more demanding for the rms of $\tilde{\delta}_k$ limited by $\frac{N}{mL^2}$. We could in principle increase $k_{hp} = \sqrt{\frac{3mH}{\hbar}}$ by increasing m to push pressure effects to the highest k-scales much beyond k_{min} and even k_{max} however high masses are in conflict with the non wrapping condition that demands m as small as possible. The only allowed way to increase m would be to increase N proportionally but this would exceed our currently available computer capacity.

Our best effort led to the following results shown in Fig 21, Fig 22 and Fig 23 for $m = 10^{-27}eV$ and then Fig 24, Fig 25 and Fig 26 for a higher mass $m = 3.10^{-27}eV$ i.e. pushing k_{hp} points to smaller scales. In both cases we check that the simulation runs as expected by looking at the growth factor at small k which remains very close to the homogeneous solution growth rate: 1.5 until reaching the NL regime. We also see that at smaller scales where pressure is non negligible the growth rate is much higher due to pressure/thermal fluctuations themselves. At still smaller scales still in the linear regime, pressure prevents the growth of fluctuations which freezes : the power spectra at various redhifts are overlapping. However this is clearly not anymore the case when the non linear regime is entered below z=3. The coupling of modes in the non linear regime (we shall clarify later what we mean here by saying non-linear even though the Schrodinger equation is linear), particularly for $10^{-27}eV$ allows many modes below and beyond k_{hp} to approach and saturate at $\delta = 1$. For modes beyond k_{hp} this behaviour is suspect: how could non linear effects allow fluctuations to grow on scales at which pressure dominates and should still prevent this growth. This behaviour is probably unphysical, only due to our limited resolution as we noticed that for a twice higher resolution simulation with N=16384 (which required 1 week of simulation with 250G of RAM in the AWS cloud) this behaviour is postponed to a lower reshift: the curves still overlap until reaching this lower redshift. The natural interpretation is that non linear coupling of modes demands still smaller scales to be resolved by the simulation for it to be reliable even on much larger scales than the resolved scales.

For $m = 3.10^{-27}eV$ without changing L, trying to push k_{hp} points to smaller scales, we had to start from significantly lower power of the ICs to avoid phase wrapping and as a result at z less than 3 we also have much less power so the entry into the non linear domain is much less advanced.

At $1.e^{-27}eV$ instead we have the satisfactory result that we do see the NL saturation however k_{hp} remains too close to k_{min}. Because of this the initial and of primordial origin assumed phase correlation at -1 between SFDM fluctuations and potentials is destroyed by pressure effects and non linear couplings on almost all accessible scales of the simulation: see Fig 22 and compare to Fig 25 where the phase correlation remains on the largest scales but we cannot be sure if this is the result of the higher mass which did not allow enough power in the ICs and therefore at the end of the simulation to clearly see the NL regime on the largest scales or if this is a proof that largest scales when these are far away enough from scales affected by pressure can enter the NL regime without losing their initial phase correlations. The latter would be expected on physical ground but we are not able to completely decide with these simulation results. Much longer simulations with more than 256G RAM would be needed for that.

Anyway we have established two important results : the NL saturation at $\delta = 1$ meaning that the fluctuations naturally end up having the same order of magnitude as on our side, and the preservation of the initial phase correlation with the potentials in the linear regime and at scales where pressure is negligible. Again, the only question which is not completely answered is if this remains the case in the NL regime: physics arguments seem to imply that away from k_{hp} both below or beyond, the NL coupling of modes with the k_{hp} mode affected by pressure and responsible for the destruction of initial phase correlations, will decrease (we already apparently see this in the power spectra) and therefore the phase correlations will be preserved but it turns out to be difficult to obtain a definite proof from our simulations.

Eventually we have strong confirmations and clues that indeed in a SFDM scenario fluctuations from the dark side could take over at the transition redshift which is mandatory to explain various void observables.

A simulation was also done for $m = 10^{-29}eV$, L=10000, so that m$\mathbf{L^2}$ is left the same as for $m = 10^{-27}eV$, L=1000 and the IC power can remain the same, and it led to very similar results except that the growth of fluctuations on pressure dominated scales is apparently slower and allows to better isolate the growth behaviour on the largest scales far from pressure dominated scales. This was then confirmed with an even lower mass run at $m = 10^{-31}eV$, L=100000 with still the same mL^2.

Now a warning and clarification are necessary regarding the effects we refer to as nonlinear. Actually the evolution of the wave function driven by the Schrodinger equation is completely linear since in our case the effect of the external potentials in the only non linear term $\Psi\Phi$ of the equation turns out to be negligible. We could indeed check that all Fourier $|\Psi_k|$ modes remain constant. The non linear effect only comes in when we eventually compute $\delta(x) = |\psi(x)|^2 - 1$, implying in Fourier space the convolution $\delta_k \sim \sum_q \psi_q \, \psi^*_{q-k}$ which dramatically amplifies the δ_k and eventually the power spectrum. The apparent nonlinear growth of the

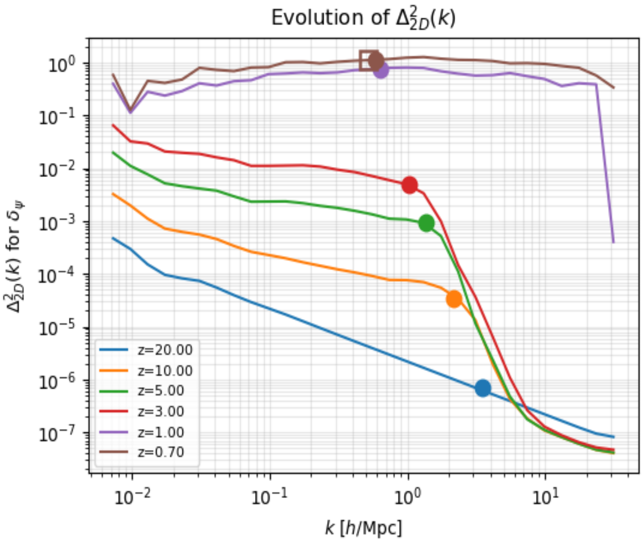

Fig. 21. SFDM adim 2d Power spectra for $m = 10^{-27}eV$. Circular dots represent k_{hp} frontier between pressure and Hubble term domination, the square represents k_{pg} between pressure and gravity

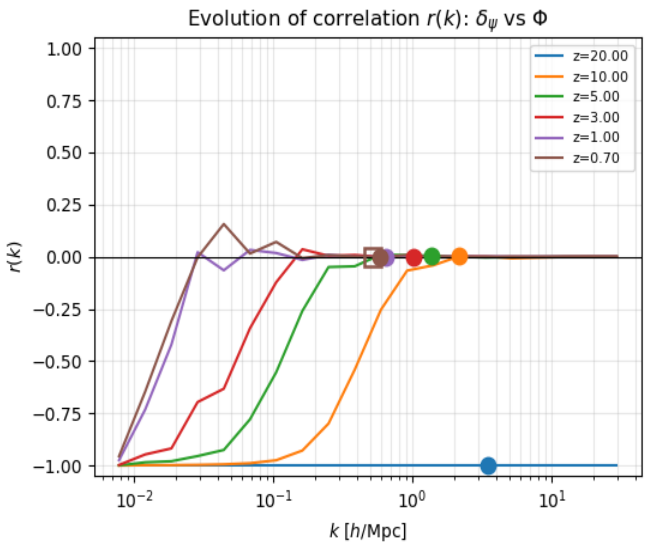

Fig. 22. SFDM Correlation with potentials for $m = 10^{-27}$eV. Circular dots represent k_{hp} frontier between pressure and Hubble term domination, the square represents k_{pg} between pressure and gravity

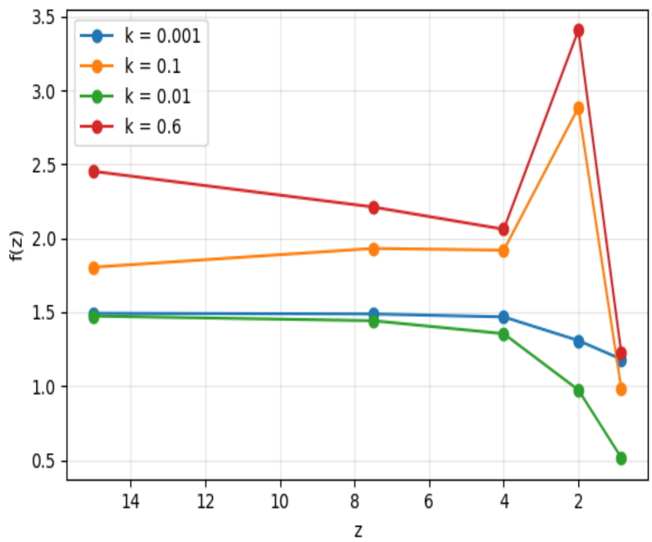

Fig. 23. SFDM Growth factor for $m = 10^{-27}$eV.

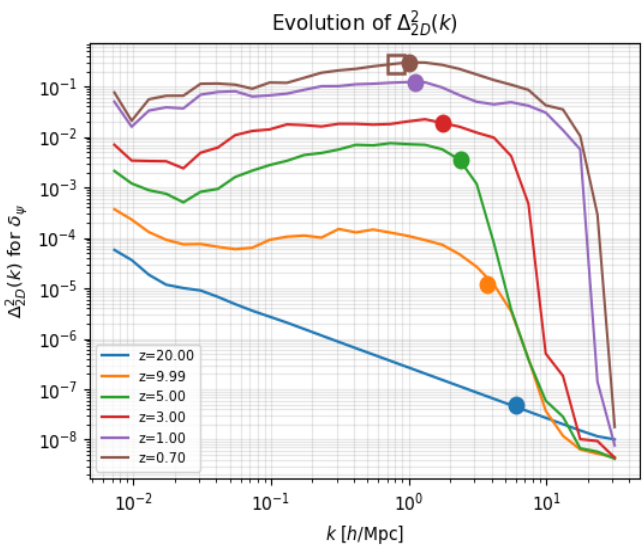

Fig. 24. SFDM adim 2d Power spectra for $m = 3.10^{-27}eV$. Circular dots represent k_{hp} frontier between pressure and Hubble term domination, the square represents k_{pg} between pressure and gravity

density field therefore arises from constructive interference between many freely evolving Schrödinger modes even on pressure dominated scales at small redshift. It

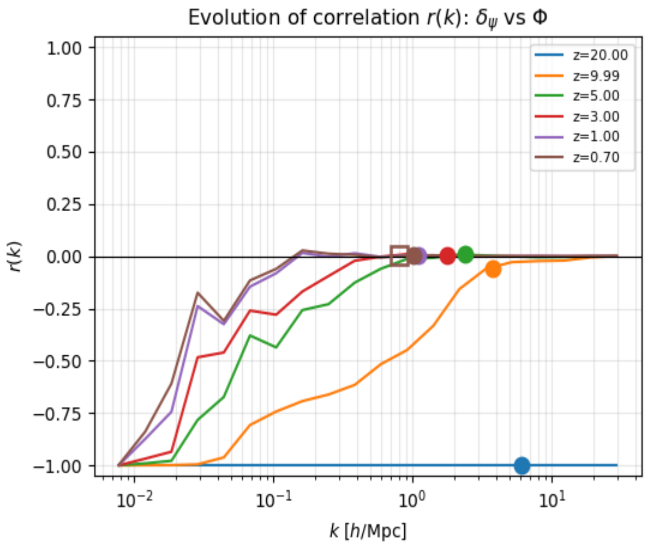

Fig. 25. SFDM Correlation with potentials for $m = 3.10^{-27}$eV. Circular dots represent k_{hp} frontier between pressure and Hubble term domination, the square represents k_{pg} between pressure and gravity

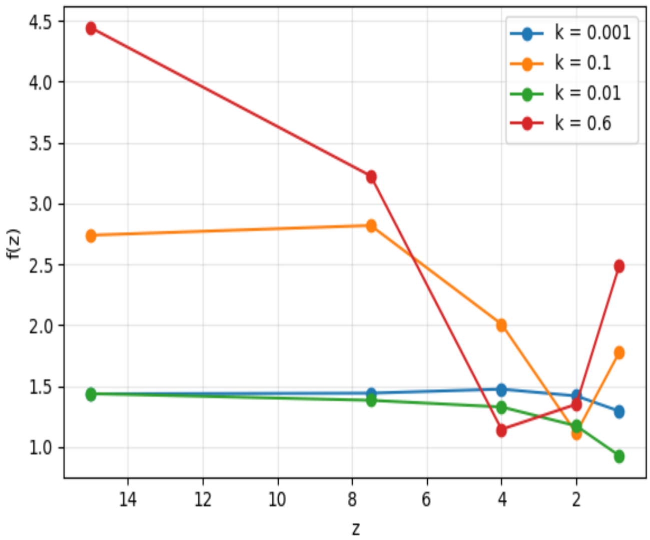

Fig. 26. SFDM Growth factor for $m = 3.10^{-27}$eV.

remains that even after applying our padding32 de-aliasing methods, $|\psi(x)|^2$ could be insufficiently resolved resulting in a nonphysical growth that we see restarting on small scales after a temporary freeze. A simulation with a twice higher N (very demanding in terms of CPU and RAM) apparently confirmed this because then the freeze appeared to be longer.

In our simulations we have assumed that $a = 1$ today, and in standard flat cosmology with only one sector, the normalisation of a is arbitrary as it is not an observable. However in DG, it really physically matters at which redshift $\tilde{a} = \frac{1}{a} = 1$. In DG this occurs by definition at the Big bang time t=0, supposed to be at a very high redshift $z0$ compared to all z we can probe by their observables.

Starting from the Dark side SFDM equation 125 in comoving coordinates, and going back to the physical coordinates of our sector $\mathbf{r} = a(t)\,\mathbf{x}$, substituting into the equation gives:

$$i\hbar\frac{\partial\tilde{\Psi}}{\partial t} = -\frac{\hbar^2 a^2}{2m}\nabla_r^2\tilde{\Psi} - \frac{m}{a^2}\Phi(\mathbf{r},t)\tilde{\Psi} - i\hbar H\,\mathbf{r}\cdot\nabla_r\tilde{\Psi}.$$

where we have used

$$\left.\frac{\partial}{\partial t}\right|_{\mathbf{x}} = \left.\frac{\partial}{\partial t}\right|_{\mathbf{r}} + H\,\mathbf{r}\cdot\nabla_r,$$

And we see that under a global rescaling of the scale factor, $a \to C\,a$, the equation remains invariant provided that $m \to C^2 m$. Therefore, if the fundamental normalization of the scale factor is chosen such that a=1 at some very early epoch instead of today, then our simulation performed with a mass m_{simu} is equivalent to a physical mass $m_{\rm true} = C^2\,m_{\rm simu}$ where C is the ratio between the new and old normalizations of the scale factor. For instance for $1 + z0 = 10^{15} = C$, the true physical mass corresponding to our $10^{-30}eV$ simulated mass is actually 1eV.

14.7. *The confrontation with growth data: σ_8, $f\sigma_8$*

14.7.1. *$f\sigma_8$ data*

The confrontation with fσ_8 data is actually almost impossible at this time given that the Alcock Packzinsky effect in the measurement of fσ_8 is even much larger than the converse effect of growth on BAO measurements. Such effect can only be reliably accounted for in a complete reanalysis of rough data assuming DG as the fiducial model instead of LCDM from the beginning. Even for models much closer to LCDM various analytical formulae which have been proposed to translate measurements obtained assuming LCDM as the fiducial model to these other models are already considered to be probably too approximate (even from Wmap LCDM to Planck LCDM : see [85]) so let alone for DG.

We nevertheless wanted to try one non trivial formula established in [84] and apply it to a model in which an effective homogeneous dark energy fluid mimics the H(z) of DG after the transition redshift. In this case the differential equations for the evolution of fluctuations have been solved numerically and solutions well reproduced by the following expressions:

$$
\begin{aligned}
z > z_{tr} \Rightarrow \sigma_8(z) &= \frac{1.06}{1+z} \\
z < z_{tr} \Rightarrow \sigma_8(z) &= 0.092z^2 - 0.469z + 0.906
\end{aligned}
\tag{131}
$$

from which follows $f\sigma_8(z) = -(1+z)\frac{d\sigma_8(z)}{dz}$ [85]. Figure 27 shows the LCDM prediction and data obtained assuming LCDM as a fiducial model in orange along with our prediction for this model (Wrong DG) and AP corrected data points from LCDM to this model in green. The fiducial model correction and predictions for DG vs LCDM (the ratio) for σ_8 and $f\sigma_8$ are also reported. For better readability the error bars are only indicated for the green points. One can see that such fiducial model correction allows the corrected data to agree as well the (wrong) DG model as the original points agreed LCDM.

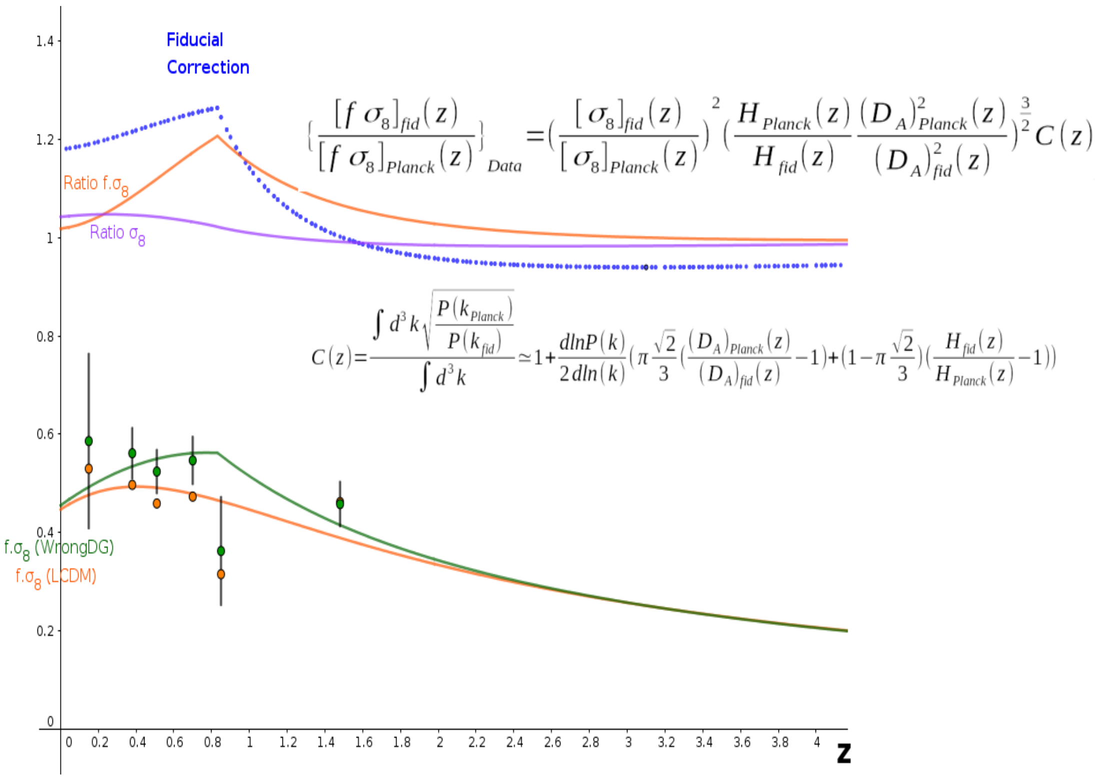

Fig. 27.

14.7.2. σ_8 *data*

This section will be revised soon following our detailed N-Body simulations. The amplitude of linear perturbations in the late universe ($z < 3$) is indirectly accessible through it's integrated effect on the Planck power spectrum of the potentials ($C_l^{\phi\phi}$). Those linear perturbations that are gravific and able to produce the lensing are our side perturbations before z_{tr} which should have grown a little bit more and reach $\approx 4\%$ more than expected within LCDM at z_{tr}. Following the transition, though the situation is more complex as for the subsequent evolution of our side fluctuations (which as we depicted in the previous section is different for under-densities and over-densities, should first evolve according a transition regime and depends on whether δ_+ is greater or less than 0.5) on which depends $f\sigma_8$, we unambiguously expect a larger lensing effect from the larger dark side linear fluctuations. This should help increase the power of the $C_l^{\phi\phi}$ on the largest scales where our approximated wrong DG model predicts a slightly too low power (see Figure 10).

There is another simple prediction that we can make for most other "σ_8" probes which actually are not able to access the effect of our side linear perturbations after z_{tr} and only very indirectly and in a limited way the effect of dark side fluctuations. Indeed, in principle, following the transition it's now the linear fluctuations of the dark side alone which should be gravific and able to produce the lensing. However those probes from which we derive σ_8 either directly measure the clustering of visible .i.e. baryonic objects: the galaxies, or measure the lensing produced by halos which are already very non linear structures. So from the point of view of DG the structures from which we claim to be able to derive the amplitude of linear fluctuations both belong to our side very non linear structures which gravity was not renormalized at the transition and which therefore retained their gravific power or at least most of it. If as is probably the case (but should be confirmed by simulations), the red frontier in Figure 12 cut out part of the total non linear structure, thereby diminishing it's gravific power, the total lensing signal from all such structures should be significantly reduced following the transition redshift and no compensation is expected from the lensing effect of dark side voids surrounding them because of their limited density contrast (-1) and small mean density of the universe just after z_{tr}. The situation may have evolved though, as we know that the dark side being in contraction should now be $1.7^6 = 24$ times denser than our side and its voids should now represent more efficient lenses though their contrast is still limited to -1. May be a great confirmation of our expectations is in [87] which from a compilation of all available data could reconstruct the evolution of S_8 as a function of redshift as shown in their figure 8. We see that with respect to LCDM expectations, the low S_8 tension is concentrated at $z < z_{tr}$. At the contrary for $z > z_{tr}$, and as we expect, S_8 is slightly exceeding expectations. The increasing S_8 at very low redshift might be the signature of the enhanced lensing from the dark side voids as the dark side universe becomes denser.

14.8. *BLK instabilities*

BKL (Belinksii, Khalatnikov, and Lifshitz)[69] instabilities are known to be generic in theories with a matter dominated contracting universe before a bounce that are interesting alternatives to inflation theories (see [66] and references therein). The catastrophic growth of anisotropic stress i.e. the universe starting to contract at different rates in the three spatial directions is a serious problem that would require an extreme suppression (hence fine tuning) of anisotropies already in the initial state of a contraction phase to avoid them growing up to an unacceptable level thereafter [70 chapter 2.2]. This becomes a concern for DG but only in the contraction phase following the transition redshift in which the dark side anisotropy "(we are not talking here about inhomogeneities but anisotropies of the background) are gravitationnally active. Fortunately this phase is preceded by our side dominated phase in which the evolution of the fluctuations on both sides is determined and driven by our side fluctuations in an expanding universe which is reducing such kind of anisotropies by exactly the same factor the next phase is expected to amplify them. Indeed our cosmological equations 5 and 6 are not modified when we allow anisotropic expansion rates. Writing the cosmological Bianchi I (still homogeneous and flat) metrics[100]:

$$d\tau^2 = a^2(t)(-dt^2 + e^{2\theta_x(t)}dx^2 + e^{2\theta_y(t)}dy^2 + e^{2\theta_z(t)}dz^2) \tag{132}$$

$$\tilde{g}_{\mu\nu} = a^{-2}(t)(-dt^2 + e^{-2\theta_x(t)}dx^2 + e^{-2\theta_y(t)}dy^2 + e^{-2\theta_z(t)}dz^2). \tag{133}$$

with $\theta_x(t) + \theta_y(t) + \theta_z(t) = 0$ we can define the two parameters $(\eta, \sigma) = (\theta_x + \theta_y, \theta_x - \theta_y)$ quantifying the anisotropies and find that it is equivalent to have our densities and pressures receiving additional new contributions $\rho_\theta = p_\theta = \tilde{\rho}_\theta = \tilde{p}_\theta = \frac{\dot{\theta}_x(t)^2+\dot{\theta}_y(t)^2+\dot{\theta}_z(t)^2}{16\pi G} = \frac{3\dot{\eta}^2+\dot{\sigma}^2}{32\pi G}$ on the rhs of our equations except that these terms do not satisfy Bianchi identities. Varying G as we did to unblock our cosmology has no effect on these new terms which were actually geometrical terms of the lhs of our cosmological equations now disguised as source terms and transferred to the rhs. But of course the game rules remain the same as we have now two new equations (the same) to be satisfied by our two new degrees of freedom (η, σ), so still one more independent equation (4 equations in total) than total 3 degrees of freedom (a, η, σ) still requiring the offshell variation of G:

$$\ddot{\eta} + 3H\frac{a^4 - 1}{a^4 + 1}\dot{\eta} = 0 \tag{134}$$

$$\ddot{\sigma} + 3H\frac{a^4 - 1}{a^4 + 1}\dot{\sigma} = 0 \tag{135}$$

Unsurprisingly $a >> 1$ implies that the anisotropies $\dot{\sigma}$ and $\dot{\eta}$ decay as $1/a^3$ and $a << 1$ that these grow as a^3 as previously anticipated, so in a perfectly symmetrical way. [n] This problem is not easy to solve in the context of bouncing universe theories as it turns out to be extremely unnatural to have a phase dominated by a field able to extremely homogeneize the anisotropies followed by a phase in which the matter field unavoidably becomes dominant (needed to get the correct spectrum of initial fluctuations after the bounce) and amplifies the anisotropies.

15. Cosmological Dark Matter reinterpretation

We already pointed out that baryonic matter is, just as within GR, cosmologically not abundant enough to account for the Hubble rate before the transition redshit, so we still need a "Dark Matter" cosmological density $\bar{\rho}_{DM}$.

It is not obvious whether this Dark matter is the key to understand the flat rotation curves of modern galaxies because as we already noticed, each galaxy is understood to produce a hole in the distribution of dark side matter, and this hole in turn behaves as a dark matter halo after the transition redshift. An amazing recent result [105] is the observation that high redshift galaxies rotation curves are apparently not flat at all, which seems to imply that most of the effects that we attribute to DM halos in the present universe are rather due to the hole in the dark side distribution because when we switch off this hole antigravity as must be the case at high redshift beyond the transition redshift, then the galaxies rotation curves fall as expected for baryonic matter only. If confirmed this is a revolution in the way we understand the behaviour and distribution of Dark Matter and the role it plays in the formation of baryonic structures. The recent JWST [106] observations (see also [107] Fig 3 and 4) of too many high redshift too massive galaxies also seem to confirm that we have a currently very bad understanding of how the first galaxies formed. So let's be open minded in listing the various possible Dark Matter candidates.

15.1. *Pseudo BH as DM candidates ?*

Primordial Black Holes (PBH) were recently considered as possible candidates for Dark Matter because these are collisionless, stable, and not completely ruled out by astrophysical and cosmological constraints as a candidate to represent the totality of Dark Matter. This is particularly the case if the objects cluster and occupy a broad range of masses allowing them to evade constraints [103] but even for a monochromatic distribution of PBHs with typical asteroid masses (from 10^{-15} to 10^{-10} solar masses) there remains an open window. Just as GR Black Holes, our pseudo BH could exist in any size and are actually submitted to the same observational constraints. We can't rely on primordial quantum fluctuations to produce them if DG cannot

[n] Notice that since we have a regular origin of time (not bringing singularities as in GR) we may exploit it to impose initial isotropic conditions $\dot{\sigma}(0) = 0$ and $\dot{\eta}(0) = 0$ or even the vanishing of the average spatial curvature as an initial condition hopefully insuring we can neglect back-reaction effects i.e. that averaged equations must then indeed take the form of equations of the averaged metric (FRW like) at any time (see the end of section 2.1 in [110]).

become a quantum theory of gravity. However we shall see in a forthcoming section that we have an alternative to inflation and quantum primordial fluctuations: in the context of DG, near t=0 thermal fluctuations may produce the initial scale invariant spectrum required by the CMB. Above threshold fluctuations could later collapse to primordial pseudo black holes whenever the equation of state drops as for instance is expected at the QCD transition around 200 MeV temperature (PPBHs are then expected to be in the solar mass range)[103]. The problem is that Gaussian initially thermal fluctuations may not be large enough to reach the threshold.

Remnant Pseudo BHs from a previous cycle are not excluded in DG and would remain an interesting alternative channel to be explored.

15.2. *Heavy elements baryonic matter as DM candidate ?*

Figure 3 from [47] summarizing all existing constraints on the existence of Macros i.e. massive Dark matter objects possibly made of standard model particles assembled in a high density object (from beyond atomic to well beyond nuclear densities) leaves open the possibility that Dark Matter could be made of condensed matter with usual atomic densities and heavy elements such as iron if this was injected from the conjugate side Pseudo Black Holes during our radiative era. Then the distribution of this injected baryonic with high metallicity DM is expected to have been extremely inhomogeneous because highly concentrated on spots, much smaller than the Planck experiment resolution, making related small scale perturbations detection hardly possible. This concentration of DM in spots with very high metallicity is needed to make the idea viable as otherwise we would hardly understand why the universe is almost everywhere we look nowadays at a very low level of metallicity (compatible with the predictions of Big-Bang nucleosynthesis and stellar nucleosynthesis) both in the diffuse intergalactic gas as well as in stars. If this hypothesis is true the corresponding high metallicity and dark regions remain to be discovered. The high metallicity is also required to insure that these nuclei have a low charge over mass ratio making them much less dragged by the primordial acoustic fluctuations and then contributing to DM rather than normal baryonic matter from the analysis of the CMB spectrum.

The serious difficulty with this DM candidate making it unlikely is that the impulse response to an initial DM perturbation at a much higher redshift than the redshift of decoupling has been studied and should produce a spreading of this DM other scales extending to tens of Mpc. So this form of DM could not have remained localized until today (as we need to understand why it evades detection) and would have been vaporized by the high temperatures unless we assume that the injection occurred at a redshift not too much higher than 1000. But then the distribution of this DM would be quite different from the one predicted within LCDM as implied by it's initial spectrum of fluctuations but also the impulse response understood to evolve as depicted in fig 1 of [52] from which we also see that it would also have influenced very differently the mass profile of baryonic matter fluctuations already at decoupling. This argument therefore seems to rule out normal matter as DM.

15.3. *Micro lightning balls as DM candidates ?*

In previous papers we also described objects called micro lightning balls (mlb) that would also be collisionless in their collapsed state (they would "decouple" from the baryon photon fluid due to their small "cross-section") and deserve much attention since these as well might be perfect Dark Matter candidates. Some of those objects, as well as pseudo BH, might have been created as the result of density fluctuations producing a gravitational potential rising above a fundamental threshold triggering the discontinuous potential trapping and stabilizing the object. Some are likely to behave as miniature stars, presumably as dense and cold as black dwarfs and extremely difficult to detect either through their black body radiation of an extremely cold object, their negligible gravitational lensing given their surface gravity much smaller than that of a pseudo Black Hole of the same size and the absence of Hawking radiation even for the smallest of these objects. Of course a much more detailed characterization of long living micro lightning balls would be needed to make firm predictions as for both their spatial and mass distribution and the best way to detect them.

The intriguing possibility that our mlbs may constitute dark matter is also again supported by figure 3 from [47]. Presumably this high density form of matter could have been injected in our universe in it's radiation dominated era (hence with a negligible influence on the scale factor evolution at this epoch) from pseudo black holes and compact stars of the dark side which was very cold at this time. This era indeed corresponds to the beginning of a contraction phase of the dark side having followed a very long lasting expansion era having resulted in a dark side universe in which most of the matter had been swallowed by Pseudo Black Holes.

The mlbs only remain a plausible candidate provided their injection occurred at a sufficiently high redshift (see our discussion of normal matter as DM candidate in the previous subsection) but not too high to avoid the destruction of mlbs by a high energy particles bombardment. These also should have been injected according a nearly scale invariant spectrum (except on the very small scales marking the initial spots) determined by the distribution of pseudo BH on the Dark Side.

Interacting with matter, mlbs can decay and release their normal matter content with presumably high metallicity in their environment. The problem with mlbs is that these require a mechanism of matter transfer between the dark side and our side: the feasibility of an extended version of DG allowing this is not granted. By the way, it is worth mentioning that discontinuities not only allow mlbs but might have helped the fast formation of stars in general and large mass ones in particular leading to many large mass pseudo BHs such as the ones recently discovered by Ligo or giant black holes at the centers of large galaxies. This is because the dragging effect of drifting discontinuities is presumably an effective mechanism to concentrate matter at all scales or to merge already formed pseudo BHs.

15.4. *Exotic hydrogen atoms as DM ?*

Nothing actually prevents the spherical discontinuity of a mlb to encompass only the simplest nucleus, the proton, to produce then an exotic hydrogen atom because the spherical symmetry imposed by the discontinuity would then forbid any non isotropic configuration of the electron cloud. Only the simplest state, the isotropic s state would logically be accessible for such hydrogen atom along with it's hyperfine levels. Then, because the only remaining possible interaction with photons would be through transitions between hyperfine energy levels, such new flavor of the hydrogen atom would behave as dark matter (being for instance almost decoupled from the primordial plasma) but would still participate in the emission and absorption of the 21cm wavelength. This is actually a funnily similar idea to the one proposed by Oks in [[89]] (the second flavor of hydrogen atoms) to explain "a 2018 perplexing observation by Bowman et al [[88]] of the redshifted 21 cm spectral line from the early Universe. The amplitude of the absorption profile of the 21 cm line, calculated by the standard cosmology, was by a factor of two smaller than it was actually observed. The consequence of thus striking discrepancy was that the gas temperature of the hydrogen clouds was actually significantly smaller than predicted by the standard cosmology. According to Mac Gaugh [[90]], "the observations by Bowman constitute an unambiguous proof that dark matter is baryonic, so that models introducing non- baryonic nature of dark matter have to be rejected". And Oks assessed that dark matter baryons, might have provided an additional cooling to the primordial hydrogen gas, a deduction which could as well apply to our own flavor version of the Hydrogen atom. As this DM candidate does not require any so far speculative mechanism of matter transfer between the dark side and our side this is our favourite one.

16. The nearly scale invariant primordial power spectrum

In the following we shall list advantages and drawbacks of some of the most popular avenues to obtain a scale invariant power spectrum before investigating the DG case. Here is a list of useful references on this topic: [[67]][[68]])[[97]][[98]][[99]][[92]][[95]][[94]][[96]][[93]].

16.1. *Inflation*

Because of the expansion laws of our universe in the radiative and matter dominated era we know that the cosmological perturbation scales presently within the Hubble radius have entered this Horizon more or less recently and we know that before this crossing time those perturbations were beyond the Hubble radius with corresponding curvature perturbations being nearly scale invariant. This approximate scale invariance (spectral index $n_s = 0.96 \pm 0.007$ hence close to 1 but with a significant 4% departure) of the primordial dimensionless power spectrum of curvature perturbations is what we have learned from the detailed study of the CMB and large scale structures.

By far the most popular theories able to produce such kind of spectra during the last two decades have been inflation theories in which fundamental scalar field quantum vacuum fluctuations initially below the Hubble radius, see, after Horizon exit, their physical scales expand faster than a quasi constant Hubble radius, thanks to a primordial inflationary phase i.e. quasi-exponential regime of the scale factor as a function of standard cosmological time resulting from the dynamics of the scalar field. Those scalar field fluctuations then freeze beyond the Hubble radius in a nearly scale invariant power spectrum also resulting in a nearly scale invariant spectrum of curvature perturbations. Such inflationary scenarios not only seemed to have the ability to reproduce the correct power spectrum of fluctuations but also could hopefully solve some of a variety of long lasting issues of standard cosmology as such theories actually were just designed to do so. However even some of the most famous initial supporters of inflation have changed their mind given that many of what was believed to be great successes of inflation have been challenged by the discovery of several serious flaws. We may actually classify cosmological issues into three categories for inflation:

- The good: issues undoubtedly solved by inflation such as:
  - the Horizon problem, an observed apparent causal connection well beyond the particle horizon within the standard model. Not only large scale homogeneity but also large scale fluctuations are evidence that interactions occurred at a time when the corresponding scales were inside the Horizon (both Hubble and particle Horizon) and inflation indeed makes this possible.
  - Scalar field quantum vacuum has almost constant in time fluctuations which should naturally generate a nearly scale invariant power spectrum at Horizon exit if inflation is allowed to start.
- The bad: Issues that are probably not solved by inflation and are the subject of intense debate among experts:
  - It was believed that inflation could explain both the early universe smoothness ($\delta_{CMB} \approx 10^{-5}$) and flatness (curvature parameter was $\approx e^{-146}\%$ 60 e-folds before present time). However it turns out that inflation could only start provided the universe was already extremely smooth and flat so that even if such conditions were fulfilled (whether this actually demands extreme fine tuning is a heavily debated topic) "inflation does not predict smoothness and flatness, it rather assumes it".
  - It is now also admitted that there is a serious multiverse problem: inflation actually predicts that most of the volume of the universe is in eternal inflation and has therefore nothing like the properties of our universe. It is very unlikely to fall in a region of the universe in which inflation has stops as we wish to reproduce the properties of the observed universe.
  - Eventually, Steinhardt argues that not only the initial conditions for inflation and the outcome (a universe compatible with what we observe) but

also the fine tuned potentials needed to make it work are extremely unlikely and asks: "if classic inflation is outdated and a failure, are we willing to accept postmodern inflation, a construct that lies outside of normal science (because of its unpredictability)? or is it time to seek an alternative cosmological paradigm ?

- The ugly: Issues for which everybody agrees that inflation does not provide a solution
  - either because the theory was not designed for that : the cosmological initial singularity implying infinite densities but also the geodesical incompleteness issue.
  - or because inflation itself relies on problematic hypothesis such as gravific initial quantum vacuum fluctuations whereas we know from the old cosmological constant problem that this is in principle ruled out by observation.

16.2. *The bouncing cosmology alternative*

This situation has motivated the exploration of many possible alternatives among which Steinhardt bouncing cosmology ekpyrotic model is one of the most popular for many reasons. Again scalar field vacuum fluctuations are the primordial seeds but now in a very slow contracting phase of the scale factor preceding the bounce thanks to a very large pressure due to an equation of state parameter $w >> 1$ for the scalar field. We can reconsider our three categories of cosmological issues in this new context.

- The good:
  - The scale factor is finite at the bounce and the reached energy scales are considered to be much below the Planck scale so that the trans-Planckian menace is avoided and trivially the cosmic initial singularity and geodesical completeness are no longer issues at least for a single bounce scenario.
  - Since, as was the case for inflation, the Hubble radius also decreases in a contracting universe it is granted that all fluctuation scales of interest have been inside this radius in the past and could evolve (the comoving Hubble radius $\mathscr{H}^{-1}$ in which $\mathscr{H} = aH$ is also the "conformal time Hubble rate", decreases with time in a contracting universe so comoving fluctuation scales $\lambda = k^{-1}$ outside the Hubble radius at the bounce must have been inside it past the bounce).
  - The particle horizon problem is also trivially solved as particles had more than enough time to interact in the contracting phase and the particle horizon could even be infinite if the contracting phase lasted an infinite time.
  - At last, perturbations (scalar gradients) and global curvature decay in a contraction phase. This would be expected generically for a $w \geq 0$ scalar field dominated contracting phase but the high pressure of the ekpyrotic scalar field performs even much better allowing these to decay much more

in the contraction phase than in the subsequent expansion phase.
- The slow contraction of the universe also allows the decay of background anisotropies (anisotropies of the expansion rate) thereby solving an issue which is generically very serious for contracting cosmologies.
- Tensor fluctuations are not produced, which is fine given that these are constrained by an observational upper bound which has already rule out the simplest inflation models.
- At last even though the transition from a contracting to an expanding regime of the scale factor requires $w < -1$, i.e. a violation of Null Energy Conditions implying a serious stability menace, the analysis has shown that the model is stable, according its authors.

- The bad :
- Less problematic but still a concern is the fact that the model can't directly produce adiabatic perturbations but needs at least two different scalar fields to get a scale invariant spectrum of entropy perturbations.
- Another not so simple mechanism is then needed to convert those into adiabatic curvature perturbations.
The sophisticated and indirect mechanisms and the very unusual equations of state for the content of the universe in the contraction phase being so different (no normal matter nor radiation) from what we have in the expansion phase makes this model less appealing according to me.
- The ugly: Perhaps the most problematic issue that remains is the fact that, as for inflationary models, a gravific quantum vacuum is needed which again sounds at odd with the present value of the cosmological constant.

16.3. *Dark gravity*

The common problem to inflationary and bouncing cosmologies is that they both rely on initial quantum vacuum fluctuations at the origin of cosmological perturbations whereas we know that vacuum energy does not gravitate. This is again the old cosmological constant problem, now back to create distrust and suspicion as for the actual validity of all models relying on initial quantum vacuum fluctuations to explain the primordial scale invariant power spectrum.

We shall realize soon (in the last section) that if gravity is not quantum as might be the case in DG may be it's not so surprising that pure vacuum graphs without any external incoming or outgoing real particle legs do not source the classical gravitational field. This simple possible solution of the old cosmological constant problem within DG of course dispels any hope of exploiting initial quantum vacuum fluctuations as seeds for primordial scale invariant fluctuations. Before looking for an alternative mechanism to get an invariant power spectrum let's first see whether Dark Gravity is in a better position to help solve all other cosmological issues.

The history of our universe according DG has much in common with a non singular bouncing universe cosmology in which an expanding phase (then the physics is almost equivalent to what we have when our side gravity dominates and drives the evolution of both background and fluctuations) followed a contraction phase (here the physics is almost equivalent to what we have when the dark side dominates and drives the evolution of both background and fluctuations).

This motivates us to define an effective scale factor $a_{eff}(t) = a(t)$ for $t > 0$ and $a_{eff}(t) = \tilde{a}(t) = 1/a(t)$ for $t < 0$. Then a_{eff} alternates between contraction and expansion with the remarkable rule that $a_{eff}(t) = a_{eff}(-t)$ $H_{eff}(t) = -H_{eff}(-t)$ (for conformal Hubble rates) so what is very specific to our case is the fact that the expansion and contraction rates and the global densities are perfectly symmetrical.

Also the transitions from contraction to expansion and vice-versa are fundamentally discontinuous in this picture. But remember that the true scale factors are $a(t)$ in eternal expansion, and $\tilde{a}(t)$ in eternal contraction as made explicit in Figure 28 in which the continuous line shows the evolution of the effective scale factor.

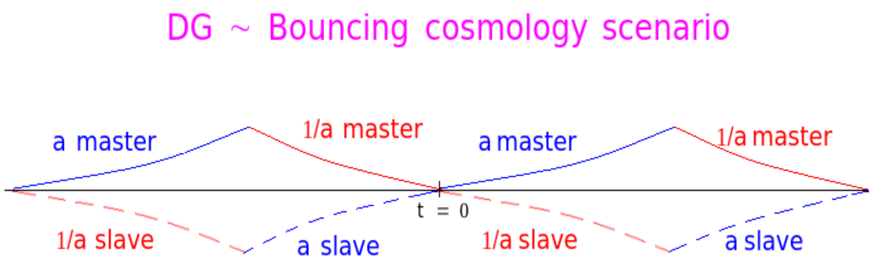

Fig. 28. DG cosmology can be reinterpreted as a bouncing cosmology with discrete transitions. The continuous line shows the evolution of the effective scale factor of the corresponding bouncing cosmology

The transition from expansion to contraction involves an efficient homogenization process. Indeed following the transition redshift it is understood that our side structures will progressively lose their gravity so that all our side structures should eventually dissipate. Of course gravity from the dark side will take over but starting from the quasi linear matter fluctuations left at the end of our side driven phase. Those fluctuations are going to grow until they freeze at potentially large values after horizon exit. Their magnitude will also depend on the time spent at the high pressure of the contracting dark side universe in the radiative era which is expected to destroy up to non linear fluctuations at scales below the Hubble radius if diffusion processes outweigh gravity even for weekly interacting Dark Matter. It is not excluded that even dark side BH candidates formed before the end of the cold contraction phase be destroyed when the background density becomes larger than the density of the very slowly collapsing star to the Schwarzschild radius (because of time dilation). But anyway fluctuations beyond the Hubble radius are frozen at potentially large values before the next transition at t=0.

Even the $\Gamma(t)$ process near t=0 which may efficiently destroy dark side fluctuations below the Hubble radius due to the very fast decay of dark side gravity it produces will have no effect on dark side density contrasts frozen beyond the Hubble radius.

The corresponding residual density contrasts δ on our side which gravity will take over, are expected to remain at most at an order of magnitude equal 1 (overdensities correspond to dark side voids bounded from below by -1 and underdensities are themselves bounded from below by -1) so, much bigger than the 10^{-5} observed in the CMB.

Let's reconsider again our three categories (in order the good, the bad and the ugly) of cosmological issues in the DG context at this level of our understanding.

- The good :
  - Our effective scale factor is finite at the effective bounce so as in any bouncing cosmology we trivially avoid issues with the trans-Planckian regime, cosmic singularities and geodesical completeness.
  - Just as in any bouncing cosmology there is in principle no difficulty to find in the contraction phase with its decreasing Hubble radius a mechanism at the origin of the CMB super-Horizon fluctuations and obviously we have no particle horizon problem.
  - Our cosmology is exactly flat by construction.
  - We should not have tensor fluctuations since we anticipate that our seeds should be thermal rather than quantum fluctuations.
  - We don't need to violate the NEC (a fluid with $w < -1$) to produce the bounce which is only effective (there actually is no bounce).
  - Even though our contraction phase is not slow, we have a perfect symmetry $a_{eff}(t) = a_{eff}(-t)$, $H_{eff}(t) = -H_{eff}(-t)$ insuring that background anisotropies are growing extremely fast in the contraction phase but just enough to completely compensate the corresponding extreme decay of background anisotropies in the preceding expansion phase. Eventually we don't expect significantly more background anisotropies than at the starting point after a full cycle (see our section devoted to BKL instabilities).
- The bad: given that we decided not to rely on initial quantum fluctuations we need to investigate the only alternative mechanism explored in the literature to hopefully produce an invariant power spectrum of density perturbations: thermal fluctuations. So far the only clue we have in favour of this origin is the fact that near t=0 we are not in vacuum conditions so thermal fluctuations are expected to dominate. But several other conditions are required to really get a scale invariant power spectrum as we shall see.
- The ugly:
  In place of scale invariant fluctuations we so far only have the fluctuations inherited from a previous contraction phase which are much larger than the ones we observe in the CMB and not scale invariant at all.

16.4. *A scale invariant power spectrum from thermal fluctuations*

Several authors have investigated the possibility that thermal fluctuations may have been the initial fluctuations. Figure 29 summarizes their computation and results.

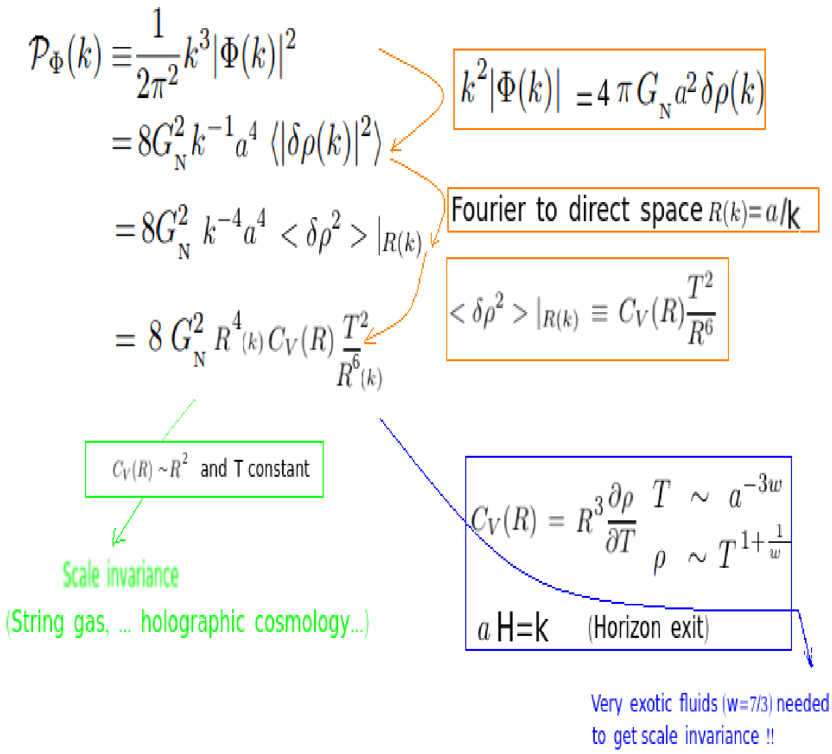

Fig. 29. The power spectrum from thermal fluctuations

The first line is the usual formula for the dimensionless power spectrum of gravity potentials $\Phi(k)$ in Fourier space which can be expressed as a function of the source fluctuation (second line) and then (third line) as a function of the fluctuation variance in direct space (in a sphere of radius $R(k) = a/k$). The latter is given for thermal fluctuations by a 19th century law in which is entering the specific heat at constant volume $C_V(R(k))$ so that eventually the power spectrum is a function of $R(k)$, $C_V(R(k))$ and the temperature T (fourth line). The computation can be carried on in the standard case in 3d (blue way) with $C_V(R) = R^3 \frac{\partial\rho}{\partial T}$ because both temperature and densities have a simple law dependency on the scale factor and the equation of state parameter w. It just remains to evaluate this at horizon crossing (aH=k) to see if for some w the k dependency in the final result could vanish. It turns out that only a very strange and exotic equation of state (w=7/3) could lead to the desired result.

A much more fascinating possibility (green way) is if the specific heat scales as a surface rather than a volume in which case R(k) terms disappear and a k dependency could only come from the temperature. Then if T is almost constant in time, nearly scale invariance is granted at Horizon exit (aH=k converts time dependency into k dependency). This is exciting because it was realized that $C_V(R) \propto R^2$ is expected in the context of various models inspired by string theory: string gas cosmology, holographic cosmology...The ideas are quite different :

In the string gas case "working under the assumption that all spatial dimensions are compact, the specific heat turns out to scale as R^2 for closed strings." This is because when the compactification radius becomes of the same order as the string length, internal string degrees of freedom are excited : the energy gradually flows into the oscillatory and winding modes of strings and as a result the temperature becomes constant and the specific heat scales as R^2. If in turn this occurs in an almost static background, the Hubble radius is quasi infinite and all scales of interest can fluctuate with a scale invariant spectrum. Those thermal fluctuations will then exit the Horizon at the end of the static phase with a small time dependency hence nearly scale invariant spectrum.

If we don't want to make appeal to strings nor compactified dimensions, the holographic principle idea according to which degrees of freedom may have been concentrated on surfaces is simpler and more suitable within the DG framework.

What is first required is of course a constant temperature and quasi infinite Hubble radius and it turns out that DG can easily provide that with a very simple extension. At t=0 the left and right hand sides of our first Friedmann-DG cosmological equation vanish whatever the conformal Hubble rate which is thus unconstrained (at the contrary $\dot{H}(t=0) = \dot{\tilde{H}}(t=0) = 0$ because of the second equation): this leaves the possibility for the two conjugate backgrounds to annihilate (because the Hubble rates are exactly opposite) and one or many new pairs of universes be recreated instantly or later with any value of $H(t=0) = -\tilde{H}(t=0)$. We then have several possible scenarios such a those pictured in Figure 30.

The most interesting possibility for us is a scenario with a static or quasi static pair of conjugate universes (green line) during T_{static} because then the Hubble radius is quasi infinite and thermal fluctuations can dissipate or prevent the growth of gravity fluctuations on scales below the Jeans length except may be large non linear fluctuations such as pseudo Black Hole candidates inherited from the previous cycle. It then just remains to understand why the energy scales as a surface during this static phase and we will at the same time get a scale invariant power spectrum of primordial thermal fluctuations and explain why most of the inherited gravitational fluctuations have been washed out by pressure. We may even explain a red tilt of the power spectrum if T_{static} was not long enough to suppress the inherited fluctuations on the largest observed scales.

Already a long time ago we understood that the interplay of positive and negative masses could produce a stable network configuration of masses with alternate signs

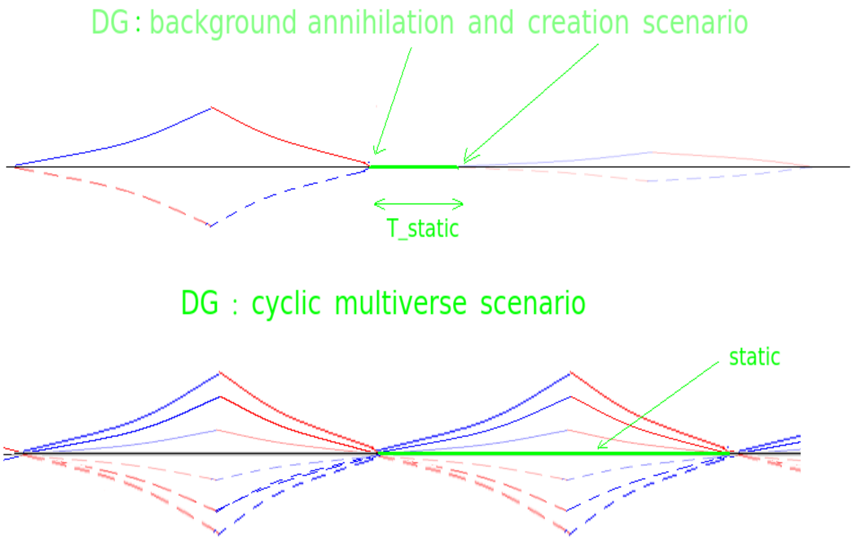

Fig. 30. Creation, annihilation or multiverse scenarios

in 3d just as atoms in a solid. These could be pseudo-BH or other less compact objects that have survived the pressure up to the end of the contraction phase on the dark side or on our side have kept a discontinuity around them to remain gravific well after the transition redshift (up to t=0). The picture that follows is shown in 2d in Figure 31.

The fluid represented in green is our side fluid repelled by dark side compact objects in the network and at the same time prevented to fall into our side compact objects by a discontinuous potential barrier around them such as the ones we have investigated in previous sections. The fluid is therefore trapped in a thin region that extends otherwise without limit. It therefore fluctuates essentially in 2d on scales much larger than the separation between network masses. Notice that there is actually a network of surfaces criss-crossing all the available volume. When the background evolution is restarted all such fluctuations find themselves instantaneously frozen beyond the horizon and then progressively stretched to cosmic scales.

It is actually unlikely that compact objects on our side will have succeeded to retain their gravity and survive up to t=0 if the discontinuity surrounding them and delimiting an asymptotically Minkowskian region enabling gravity to retain its strength within it, has finished it's centripetal drift toward a center eventually making the region disappear there. On the other hand the same kind of delimited asymptotically Minkowskian regions formed on the dark side during the dark side dominated phase are not menaced to disappear except by the high pressure reached when approaching t=0. Even the decay of $\tilde{G} = 1/G$ near t=0 should not affect them

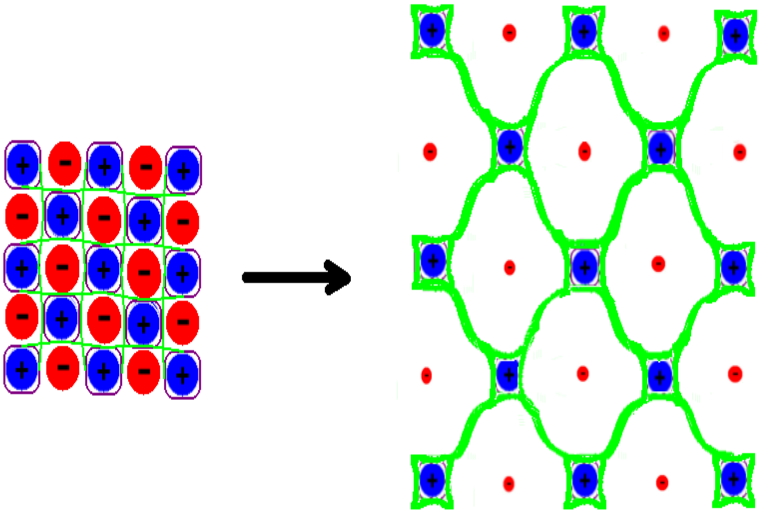

Fig. 31. A network of alternate mass signs in a solid like 3d configuration. After expansion the negative masses are represented smaller to account for the decoupling of the dark side. Magenta lignes about positive masses represent discontinuous potential barriers and green lignes represent the trapped fluid, After expansion these lignes are thicker and will eventually fill almost all the available volume after they reenter the horizon but their fluctuations should remain scale invariant on larger scales

but only the cosmological domain all around. The corresponding picture (artistic view), starting from t=0 in the static phase, would then rather be this one 32:

17. An alternative postulate for the DG transition

So far our fundamental postulate for the permutation of scale factors responsible for our cosmological transition, has been that it is a unic event triggered by the equality of our side and dark side densities when they cross each other at z_{tr}. An alternative we want to explore in the near future is that actually many such permutations might have occurred near z_{tr} at a high rate compared to cosmological time scales, following a probabilistic rule : the probability at any time that $a > \tilde{a}$ is given by $\frac{<\rho>}{<\rho>+<\tilde{\rho}>}$ instead of 1 (resp 0) before (resp after) the transition redshift now defined by $<\rho>=<\tilde{\rho}>$. where quantities in $<>$ are understood to be averaged in time or equivalently z over a short range $[z-\delta z, z+\delta z]$ compared to cosmological scales. Yet this time interval is long enough to host many permutations that are triggered at a very fast rate according the previous law just because ρ and $\tilde{\rho}$ fluctuate very much and therefore cross each over many times as we shall see (remember that for symmetry reasons, the crossing condition $\rho = \tilde{\rho}$ is mandatory for a permutation to occur). The reason why instantaneous densities are able to fluctuate at a fast rate around their time average $<\rho>, <\tilde{\rho}>$ is the moving internal frontier of our

Fig. 32. A variation on the same idea: A network of "negative energy masses" (from dark side in red) with our side "positive energy" fluid in between (in blue) dominated by thermal fluctuations.

cosmological domain: a short period cyclic drift according our sections devoted to the Pioneer effect. Of course this cyclically moving frontier implies a cyclic variation of the cosmological densities which is not accounted for in the usual cosmological equations without frontier.

With this new postulate, eventually at any redshift z the time averaged effective equation of state w will be a mixture of w=0 (our side matter drives the evolution) and w= -2/3 (the dark side matter drives the constantly accelerated law) with weights determined by relative cosmological densities. This should eventually produce a smooth LCDM like transition between an asymptotically w=0 in the past and w=-2/3 (rather than -1 for LCDM) in the future which might better fit cosmological observables.

Another major advantage of this approach is that it might make it natural for photons to always travel in the "same metric as gravitational waves" if they are pulled towards the metric with the higher scale factor when a scale factor permutation takes place which is ideal to explain the coincidence between GW170817 and it's associated GRB. This is actually one of the two explanations we already proposed for this coincidence but the new multi-permutation postulate we adopt now makes it more natural and trustworthy.

18. Last remarks and outlooks

18.1. *Which fundamental constants should actually vary ?*

Our cosmology looks cyclic in the purely gravitational sector as a result of the offshell variation of G: in particular densities are back to their initial values at the end of a full cycle. But this variation does not compensate the fact that expansion and contraction of the scale factors have also effects in the non gravitational sector. So the question is : do we really want all the physics to be as much as possible close to a cyclic scenario, in which case an additional variation of the Planck constant h is better suited than a variation of G alone to compensate as much as possible all expansion or contraction effects over a full cycle and this not only in the gravitational sector...or should we completely give up the cyclic picture, and adopt for instance an eternal static universe before and after the expansion and contraction episode? Investigations are ongoing.

18.2. *Frame dragging and gravitational wave anomalies?*

Earlier in this article we considered what we called a scalar-η field and investigated the consequences of having such a solution plus perturbation instead of the full metric with all it's degrees of freedom in the radiative era. We saw that such field would lead to anomalies such as the absence of gravitational waves but also frame dragging effects. We also noticed that a C=1 domain would also have almost vanishing gravitational waves solutions. At last, in some static domains cut out of the rest of the expanding universe, we might also have local rotating preferred frame attached to a rotating body with respect to the universe such that frame-dragging would also vanish in the vicinity of such body. So the DG theory asks us to seek many kinds of possible gravitational anomalies which are not absolutely excluded a priori in many different contexts and that could even be transient. In this spirit, we are tempted to interpret the zero frame dragging effect which was initially observed by Gravity Probe B on one of its four gyroscopes as evidence for DG. See our section 12 devoted to gravitomagnetism and preferred frame effects in [4] for further details.

18.3. *Status of the Janus field*

We already pointed out that none of the faces of our gravitational Janus field could be seriously considered as a candidate for the spacetime metric. Yet, though the gravitational field loses this very special status (be the spacetime metric) it had within GR, it acquires another one which again makes it an exceptional field : it is the unic field that makes the connection between the positive and negative energy worlds (this definition is relative: for any observer the negative field is the one that lives on the other side), the only one able to couple to both the dark side SM fields and our side SM fields. This special status alone implied that the gravitational interaction might need a special understanding and treatment avoiding it to be quantized as the other interactions. Avoiding ghost instabilities related to the infinite phase space opened by any interaction between quantum fields that do not carry energies with the same sign, is a requirement that might prevent the Janus field to interact with matter as a quantum field in Eq (1) and (2) . So the old question whether it is possible to build a theory with a classical gravitational field interacting with all other fields being quantum, was back to the front of the stage just because the usual answer "gravity must be quantized because everything else is quantum" fails for the Janus theory of the gravitational field.

18.4. *Gravity of quantum fluctuations*

Another point that deserves much attention is that within DG, wherever the two faces of the Janus field are equal, vacuum energy terms trivially cancel out as we already noticed in [15] so we might have good reasons to suspect that a mechanism is at work to insure that this cancellation is preserved even when the two faces depart from each other. First, cosmological constant terms are strictly constant within GR because of the Bianchi identities which is not necessarily the case in DG. Such terms might vary (because of varying cutoffs for instance) in order to preserve the cancellation between our side and the dark side vacuum energy terms. The context is anyway much more favourable than within GR where no such kind of cancellation could possibly occur.

Moreover the old cosmological constant problem is not necessarily a concern for a semi-classical theory of gravity as is DG. Indeed, the usual formulation of the problem is that we have no reason to doubt the existence of vacuum Feynman graphs since we see their effect for instance through the Casimir and Lambshift effects. However, it should be specified that the actual Feynman graphs probed this way have external legs of particles so that the extrapolation to gravity becomes straightforward: we just need to replace those external particles by gravitons to estimate how much gravity we can expect from such quantum fluctuations. The extrapolation is far less trivial when we don't have gravitons, as we should replace in this case the external particle legs by new kinds of legs actually representing an external classical field. The problem then is that the purely quantum part of the graph is really a vacuum graph: it has no external legs and we don't have any evidence that such graphs without any real particle actually exists: in particular it's not the kind of graph probed by the Lambshift and Casimir effects. Eventually the old cosmological constant problem might already be a strong clue that gravity is not quantum.

18.5. *Discrete symmetries, discontinuities and quantum mechanics*

Dark Gravity from the beginning is a theory involving both discrete (a permutation symmetry now understood as a global time reversal) and usual continuous symmetries unified in the same framework thanks to the crucial role payed by our background non dynamical metric.

It was then natural to wonder whether such discrete symmetry could have a genuine dynamical role to play, and we postulated a global time reversal process exchanging the two faces of our Janus field [7][14][15] and producing field discontinuities at the frontier of space-time domains. We now then have a unique and remarkable framework unifying not only the discrete and continuous symmetries but also the related continuous and discontinuous processes. Continuing to build on our successes we later considered various new possible discrete physical laws i.e. we may not only have discontinuous transitions in time when the conjugate scale factors exchange their roles but also other kind of discontinuities in space at the frontier between static and expanding spatial regions. We did not encounter any serious obstacle proceeding along this way and for instance we already drew the reader attention to the harmlessness of discontinuous potentials as for the resolution of wave function equations in the presence of discontinuities. Of course the exploration of this new physics of discontinuities in relation to discrete symmetries is probably still at a very early and fragile stage and requires an open minded effort because it obviously questions habits and concepts we used to highly value as physicists.

Discontinuous fields also put into question the validity of the Noether theorem implying the violation of local conservation laws wherever the new physics rules apply. However, we should remind ourselves that the most fundamental postulates of quantum physics remain today as enigmatic as they appeared to physicists one century ago: with the Planck-Einstein quantization rules, discontinuous processes came on to the scene of physics as well as the collapse of a wave function taken at face value obviously implies a violation of almost all local conservation laws.

Based on these facts, a new theoretical framework involving a new set of discrete and non local rules which, being implied by symmetry principles are not anymore arbitrary at the contrary to the as well discontinuous and non-local quantum mechanics postulates, might actually be a chance. A real chance indeed as they open for the first time a concrete way to hopefully derive the so arbitrary looking quantum rules from symmetry principles and may be eventually relate the value of the Planck constant to the electrical charge, in other words compute the fine structure constant. We are certain that only our ability to compute the fine structure constant would demonstrate that at last we understand where quantum physics comes from rather than being only able to use it's rules like a toolbox. With the classical discontinuous field of DG we are confident that we are much closer to establish a connection with the quantum fields than ever before. Again the unification of the continuous and the discontinuous seems to us a much more fundamental goal than simply trying to make gravity work with quantum rules if the later remain completely enigmatic.

In this perspective, it may be already meaningful to notice that our Pseudo Black Hole speculated discontinuity at the pseudo horizon, which would lie at the frontier between approximate GR and DG C=1 domains, behaves as a wave annihilator for incoming GW waves and a wave creator for outgoing waves. In the DG C=1 domain, the waves carry almost no energy while in the GR domain they carry energy and momentum as usual. This is a fascinating remark because this would make it the only known concrete mechanism for creating or annihilating waves à la QFT or even a significant step toward a real understanding of the wave function collapse i.e. in line with a realistic view of quantum mechanics. Such collapse is indeed known to be completely irreducible to classical wave physics because it is non local, and in fact just as non local as would be a transition from GR $C >> 1$ to DG, C=1 in the inside domain.

18.6. *The Janus field and the Quantum*

In the previous section we emphasized our theoretical motivation for bridging the gap between our classical discontinuous Janus field and true quantum fields. We also have now an additional phenomenological motivation: the old cosmological constant problem might just disappear if gravity is classical.

We already mentioned semi-classical gravity as a candidate theory to describe the interactions between usual quantum fields and a classical gravitational field. The idea is to have the expectation value of the energy-momentum tensor rather than the tensor itself sourcing the gravitational field equations. In vacuum we would need to modify the QFT formalism (if possible) to insure that true vacuum graphs (without any external real particle legs) give no contribution to this expectation value. More specifically such modification would be motivated by the idea that it is the external real particle legs that decide which metric all particles in the graph are propagated in. In the absence of external particles (true vacuum graph) the graph particles presumably remain on the Minkowski background metric insuring that such graphs will never contribute to the vev and related cosmological constant terms always vanish.

One often raised issue with semi-classical gravity is that it is incompatible with the Multi Worlds Interpretation (MWI) of QM since within the MWI the other terms of quantum superpositions which are still alive and represent as many parallel worlds would still be gravific as they contribute to the energy momentum tensor expectation value and should therefore produce large observational effects in our world. The MWI, considered as a natural outcome of decoherence is adopted by a large and growing fraction of physicists mainly because is considered the only alternative to avoid the physical wavefunction collapse. For this reason incompatibility with the MWI is often deemed prohibitive for a theory. Since we have nothing against a physically real wave function collapse (our theory even has opened new ways to hopefully understand it; discontinuity and non locality are closely related) we are not very sensitive to the incompatibility between semi-classical gravity and the MWI. The wave function collapse might eventually be triggered at the gravitational level: a simple achievement of something similar to the Penrose idea (gravitationally triggered collapse) seems within reach in our framework, thanks to a transition to C=1 which is tantamount to a gravitational wave collapse.

So we can still alternatively consider semi-classical gravity and the Schrodinger-Newton equation it implies [39] in the context of a true physical collapse interpretation of QM, all the more so that the usual arguments based on the measurement theory often believed to imply that gravity must be quantized have recently been re-investigated in [38] (see also [81]) and the authors to conclude that "Despite the many physical arguments which speak in favor of a quantum theory of gravity, it appears that the justification for such a theory must be based on empirical tests and does not follow from logical arguments alone." This has even reactivated an ongoing research which has led to experiment proposals to test predictions of semi-classical gravity, for instance the possibility for different parts of the wave functions of a particle to interact with each other non linearly according classical gravity laws. However "together with the standard collapse postulate, fundamentally semi-classical gravity gives rise to superluminal signalling" [38] so the theoretical effort is toward suitable models of the wavefunction collapse that would avoid this superluminal signalling. From the point of view of the DG theory this effort is probably unnecessary because superluminal signalling would not lead to inconsistencies as long as there exists a unic privileged frame for any collapse and any instantaneous transmission exploiting it. We indeed have such a natural privileged frame since we have a global privileged time to reverse, so it is natural in our framework to postulate that this frame is the unic frame of instantaneity. Then the famous gedanken experiments claimed to unavoidably lead to CTCs (Closed Timelike Curves) do not work any more : the total round trip duration is usually found to be possibly negative only because these gedanken experiments exploit two or more different frames of instantaneous signaling. Let's be more specific : Does instantaneous hence faster than light signalling unavoidably lead to causality issues? : apparently not if there is a single unic privileged frame where all collapses are instantaneous. Then i (A) can send a message to my colleague (B) far away from me instantaneously and he can send it back to me also instantaneously still in this same privileged frame using QM collapses (whatever the relative motions and speeds of A and B and relative to the global privileged frame): the round trip duration is then zero in this frame so it is zero in any other frames according special relativity because the spatial coordinates of the two end events are the same: so there is no causality issue since there is actually no possible backward in time signalling with those instantaneous transmissions... in case there is some amount of time elapsed between B reception and re-emission, eventually A still receives it's message in it's future: no CTC here.

It remains to be investigated whether other difficulties of semi-classical gravity already well known in the context of GR ([71]) among which bad non linearities, divergences and instabilities, remain in the context of DG. A collapse of the quantum fields should also result in a discontinuous non-local behaviour of the energy-momentum tensor vacuum expectation value, hence of the corresponding gravitational field which is not acceptable because the Bianchi identities satisfied by the classical gravitational field (rigorously in GR, and in a very good approximation in DG) are local conservation equations. Actually standard QM involves discontinuous behaviour of the fields at two different apparently unrelated levels, first the quantum vertices involving the obviously discontinuous creation and annihilation of waves, and second the collapse of wave packets. Semi-classical gravity (the vev prescription at the source of the equations of gravity) already eliminates the first kind of discontinuous behaviour at the source and the related menace to the Bianchi identities. For the second menace represented by the genuine collapse of the wave function the solution is that DG differential equations are only piecewise valid, i.e. in some delimited space-time domains while the discontinuous collapses are better understood as processes taking place at the frontier of such domains. Obviously if nature is from time to time fundamentally discontinuous and non local it cannot satisfy the continuous and local equations at the same time, rather the two phenomena must have their own non overlapping domains of definition. In a certain sense even the QFT description with it's succession of vertices and propagators already satisfies this mandatory requirement.

18.7. *Closed timelike curves*

At last, the issue of CTCs (closed timelike curves) is worth a few more words: in the context of GR it is known that a necessary condition to avoid CTCs is to ban negative energies at the source of Einstein equation (Hawking theorems). It is therefore interesting that in the limit of infinite C, in which DG tends to GR, negative energy terms also tend to decouple at the source. It is therefore left as an open mathematical problem whether for finite C values, the modification of the geometrical part of DG equations vs Einstein equations is just what we need to still avoid CTCs even in presence of negative energy source terms.

19. Conclusion

New developments of DG not only solve the tension between the oldest version of the theory and gravitational waves observations but also provide a renewed and reinforced understanding of the Pioneer effect as well as the recent cosmological acceleration. An amazing unification of MOND and Dark Matter phenomenology seems also at hand. The most important theoretical result remains the avoidance of both the Big-Bang singularity and Black Hole horizon.

Appendices

A. Field equations derivation

To get our field equation we demand that the action variation δS should vanish under any infinitesimal variation $\delta g_{\mu\nu}$. But the variation of $g_{\mu\nu}$ implies a variation of $\tilde{g}_{\mu\nu}$ resulting in the following variation of the total action integrand which must vanish:

$$\sqrt{g}(G^{\mu\nu} + 8\pi G T^{\mu\nu})\delta g_{\mu\nu} + \sqrt{\tilde{g}}(\tilde{G}^{\mu\nu} + 8\pi G \tilde{T}^{\mu\nu})\delta \tilde{g}_{\mu\nu} = 0 \qquad (136)$$

The variations are related by

$$\delta \tilde{g}_{\mu\nu} = \eta_{\mu\rho}\eta_{\nu\sigma}\delta g^{\rho\sigma} = -\eta_{\mu\rho}\eta_{\nu\sigma}g^{\rho\tau}g^{\sigma\kappa}\delta g_{\tau\kappa} \qquad (137)$$

since the Minkowski metric not being dynamical, does not vary. Replacing in 136, we get :

$$\sqrt{g}(G^{\mu\nu} + 8\pi G T^{\mu\nu})\delta g_{\mu\nu} - \sqrt{\tilde{g}}(\tilde{G}^{\mu\nu} + 8\pi G \tilde{T}^{\mu\nu})\eta_{\mu\rho}\eta_{\nu\sigma}g^{\rho\tau}g^{\sigma\kappa}\delta g_{\tau\kappa} = 0 \qquad (138)$$

Or, after a convenient renaming of the indices $(\mu, \nu) \leftrightarrow (\tau, \kappa)$ in the second term:

$$\left[\sqrt{g}(G^{\mu\nu} + 8\pi G T^{\mu\nu}) - \sqrt{\tilde{g}}(\tilde{G}^{\tau\kappa} + 8\pi G \tilde{T}^{\tau\kappa})\eta_{\tau\rho}\eta_{\kappa\sigma}g^{\rho\mu}g^{\sigma\nu}\right]\delta g_{\mu\nu} = 0 \qquad (139)$$

The resulting single equation of motion can be reshaped in a more elegant form multiplying it by $\eta^{\delta\lambda}g_{\delta\mu}$, and using $\eta_{\kappa\sigma}g^{\sigma\nu} = \eta^{\sigma\nu}\tilde{g}_{\sigma\kappa}$ (inverse metrics).

$$\sqrt{g}(G^{\mu\nu} + 8\pi G T^{\mu\nu})\eta^{\delta\lambda}g_{\delta\mu} - \sqrt{\tilde{g}}(\tilde{G}^{\lambda\kappa} + 8\pi G \tilde{T}^{\lambda\kappa})\eta^{\sigma\nu}\tilde{g}_{\sigma\kappa} = 0 \qquad (140)$$

Of course this field equation is invariant under the permutation of F and $\tilde{F}$ fields (both metrics and matter-radiation fields) just as the action we started from. We can also contract the term in square brackets in (139) with $g_{\mu\nu}$ to get:

$$\sqrt{g}R - \sqrt{\tilde{g}}\tilde{R} = 8\pi G(\sqrt{g}T - \sqrt{\tilde{g}}\tilde{T}) \qquad (141)$$

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